Physics

Todays Agenda

- Reference frames and relative motion.
- Uniform Circular Motion

Inertial Reference Frames:

- A **Reference Frame** is the place you measure from.
  - It's where you nail down your (x,y,z) axes!
- An Inertial Reference Frame (IRF) is one that is not accelerating with respect to the " fixed stars".
  - We will consider only IRF's in this course.
  - The earth is a pretty good IRF even though it is accelerating (rotating) w.r.t. the stars slightly.
- Valid IRF's can have fixed velocities w.r.t each-other.
  - More about this later when we discuss forces.
  - For now, just remember that we can make measurements from different vantage points.

Relative Motion

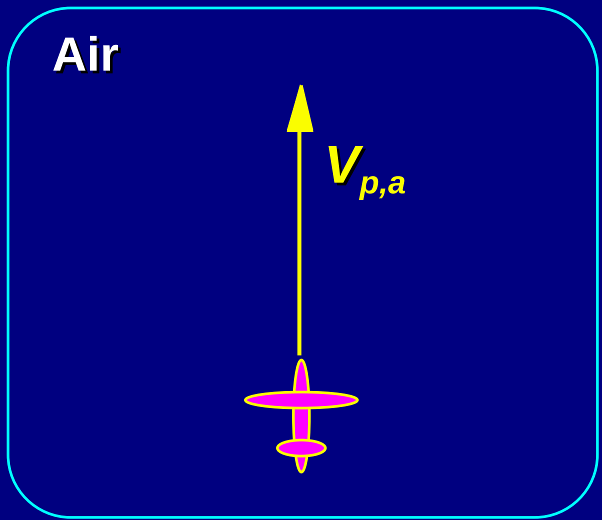
- Consider a problem with **two** distinct IRF's:
  - **An airplane flying on a windy day.**

A pilot wants to fly from Austin to Dallas. Having asked a friendly student, she knows that Dallas is 120 miles due north of Austin. She takes off from the Airport at noon. Her plane has a compass and an air-speed indicator to help her navigate.

- The compass allows her to keep the nose of the plane pointing north.
- The air-speed indicator tells her that she is traveling at 120 miles per hour **with respect to the air**.

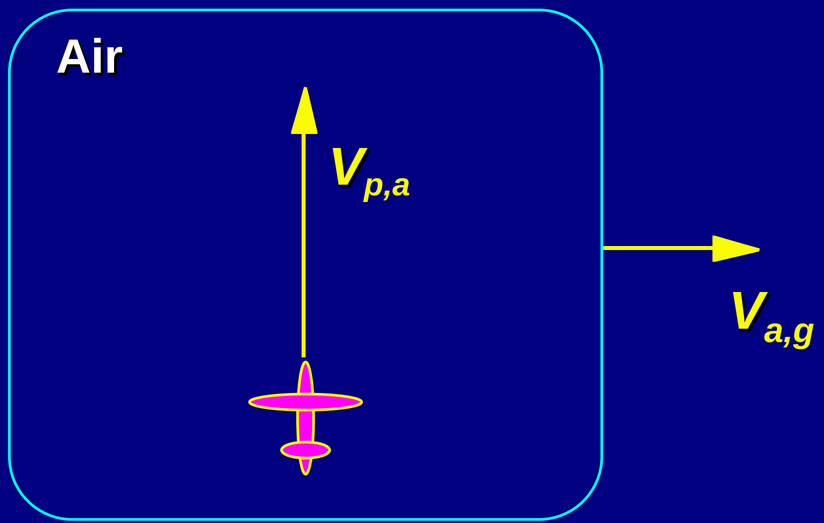
Relative Motion...

- The plane is moving north in the IRF attachd to the air:
  - $V_{p,\,a}$ is the velovity of the plane w.r.t the air.



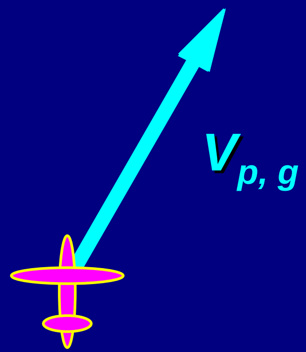
Relative Motion...

- But suppose the air is moving *east* in the IRF attached to the ground.
  - $V_{a,g}$ is the velocity of the air w.r.t the ground (i.e. *wind*).



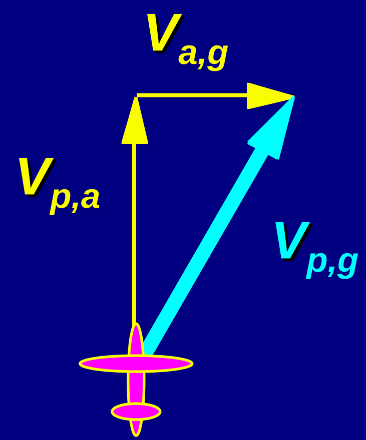
Relative Motion...

- What is the velocity of the plane in an IRF attached to the ground?
  - $V_{p,\ g}$ is the velocity of the plane w.r.t the ground.

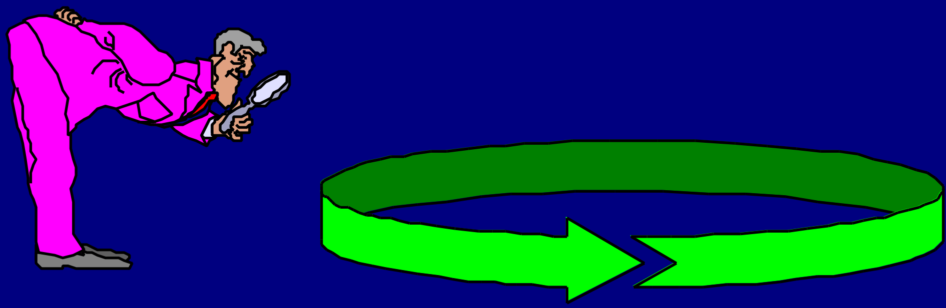


Relative Motion...

$V_{p,g} = V_{p,a} + V_{a,g}$ Is a vector equation relating the airplanes velocity in different reference frames.



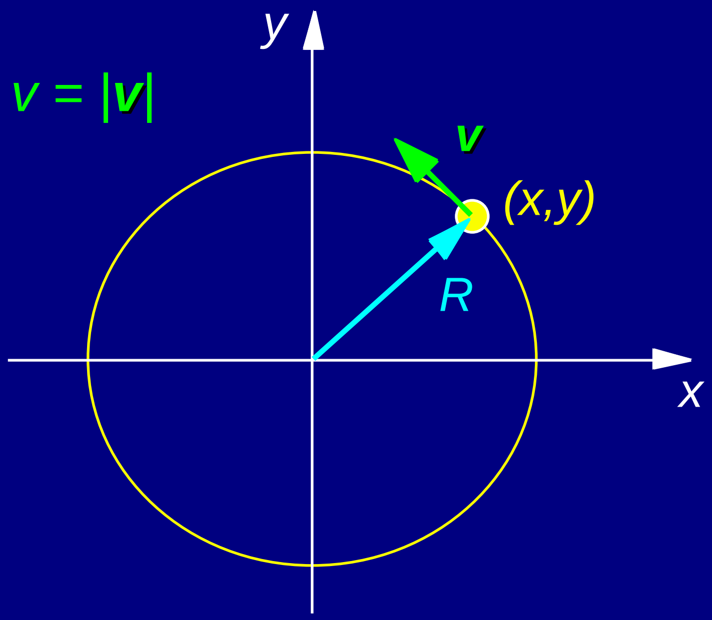
Uniform Circular Motion



- What does it mean ?
- How do we describe it ?
- What can we learn about it ?

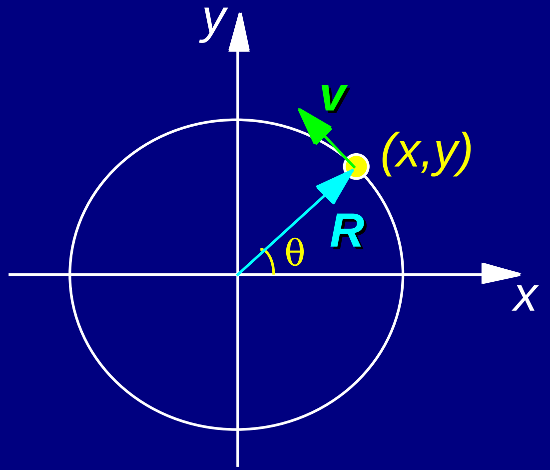
What is UCM?

- Motion in a circle with:
  - Constant Radius *R*
  - Constant Speed $v = |\mathbf{v}|$



How can we describe UCM?

- In general, one co-ordinate system is as good as any other:
  - Cartesian:
    - *(x,y)* [position]
    - *(v_x, v_y)* [velocity]
  - Polar:
    - *(R,θ)* [position]
    - *(v_R, ω)* [velocity]



- In UCM:
  - *R* is constant (hence $V_R = 0$).
  - ω (anguar velocity) is constant.
  - Polar co-ordinates are a natural way to describe UCM!

Polar Coordinates:

- The arc length ***s*** (distance along the circumference) is related to the angle in a simple way:

  $s = R\theta$**,** where θ is the *angular displacement*.

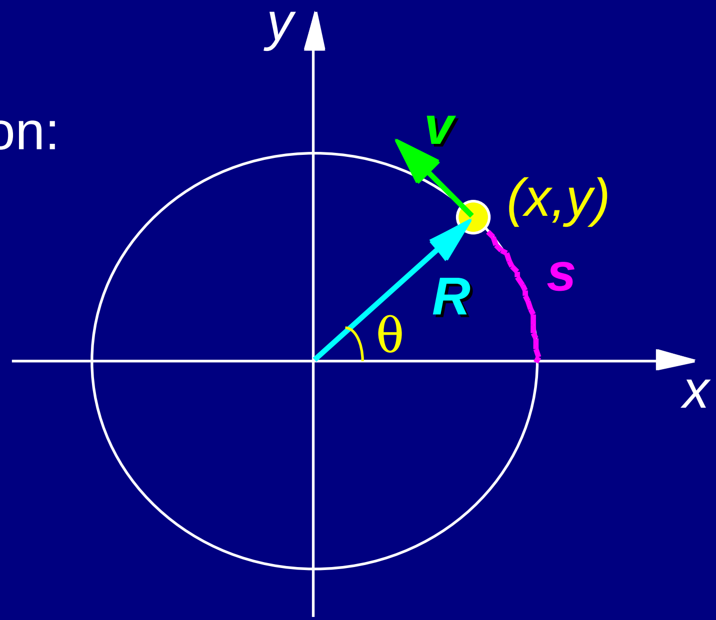
  ⇒ units of θ are called *radians*.

- For one complete revolution:

  $2\pi R = R\theta_c$

  ⇒ $\theta_c = 2\pi$

  θ has *period* 2π.

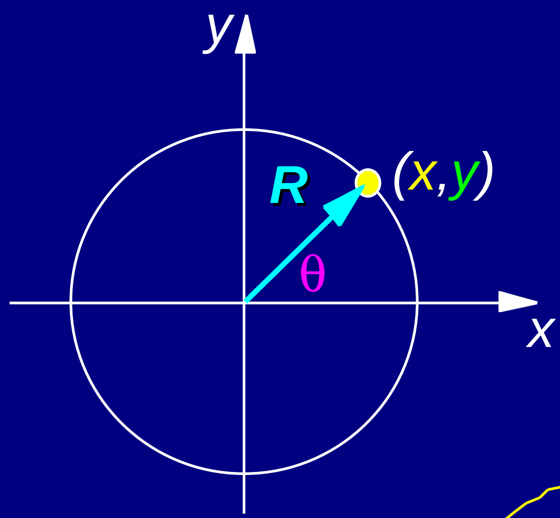


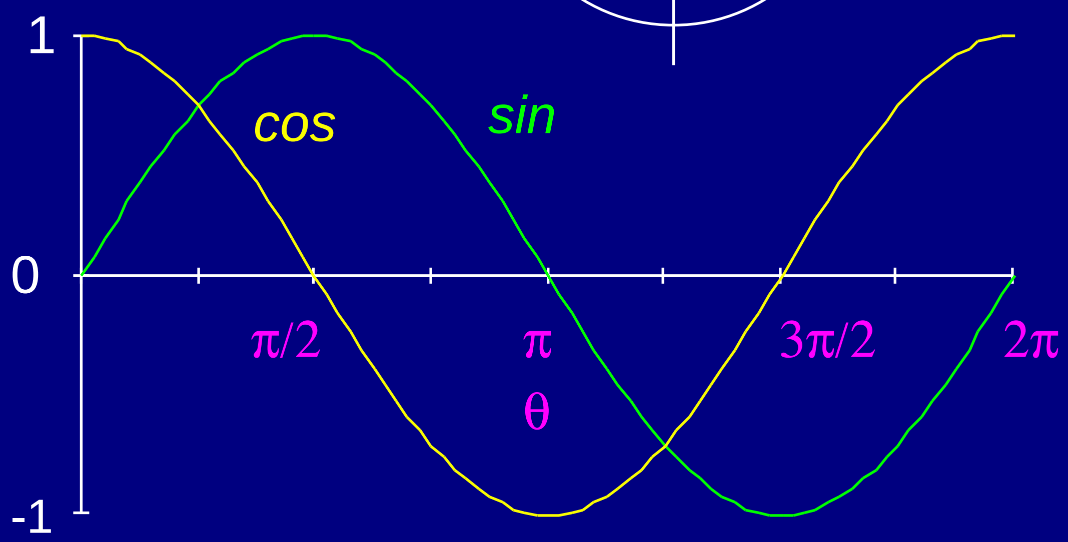
⇒ 1 revolution = 2π radians.

Polar Coordinates...

$x = R\cos(\theta)$

$y = R\sin(\theta)$

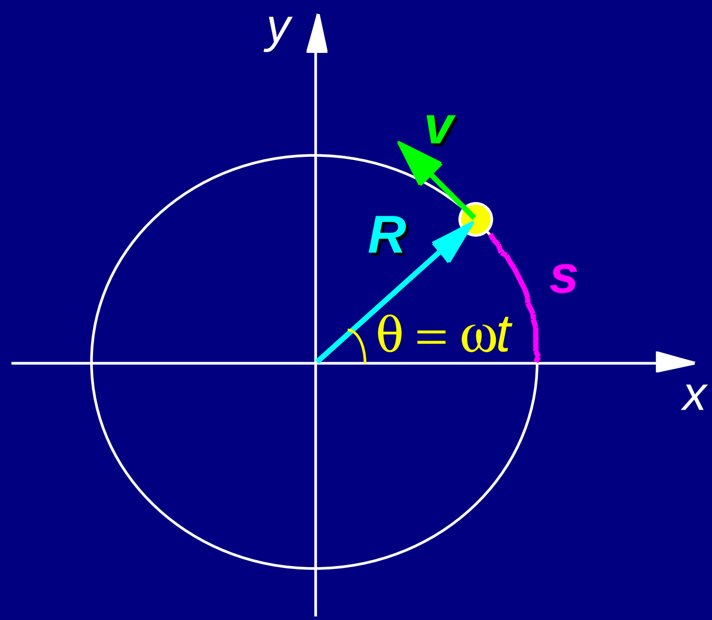




Polar Coordinates...

- In cartesian co-ordinates we say velocity $dx/dt = v$.
  - $x = vt$
- In polar coordinates, angular velocity $d\theta/dt = \omega$.
  - $\theta = \omega t$
- Displacement $s = vt$.

  but $s = R\theta = R\omega t$.
  - so $v = R\omega$

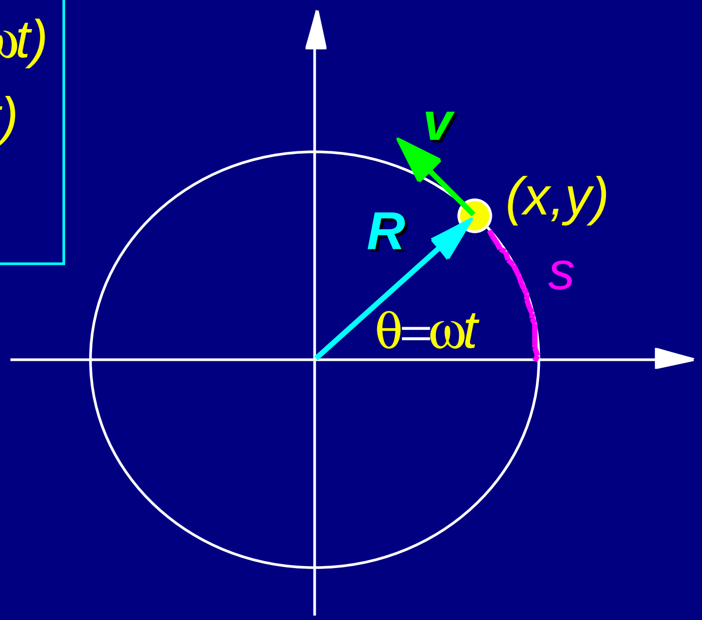


Recap:

$x = R\cos(\theta) = R\cos(\omega t)$

$y = R\sin(\theta) = R\sin(\omega t)$

$\theta = \tan^{-1}(y/x)$



$\theta = \omega t$

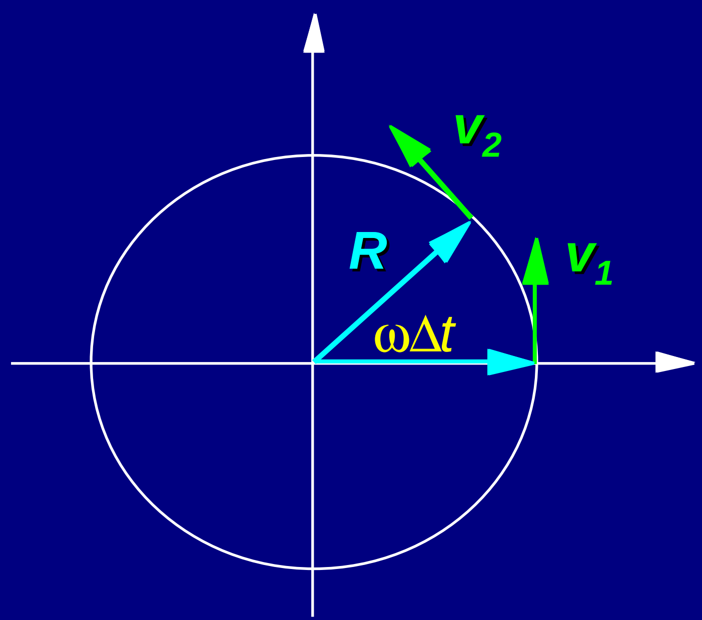
$s = vt$

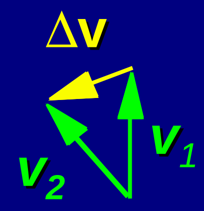
$s = R\theta = R\omega t$

$v = R\omega$

Acceleration in UCM:

- Even though the *speed* is constant, velocity is ***not*** constant since the direction is changing.
  - Consider average acceleration in time Δt ⇨ $\Delta \mathbf{v} / \Delta t$





Acceleration in UCM:

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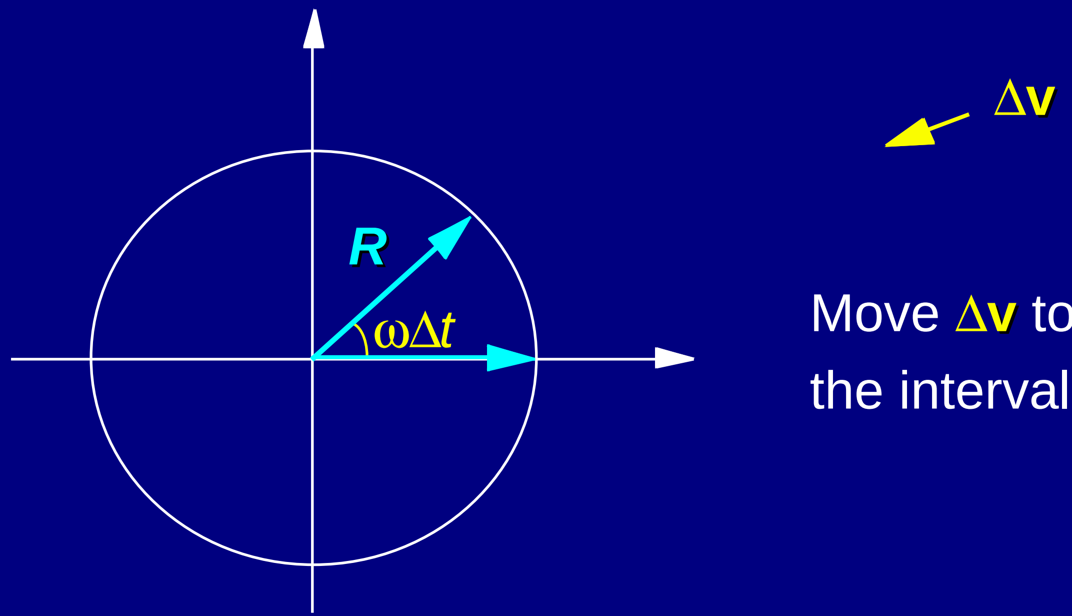
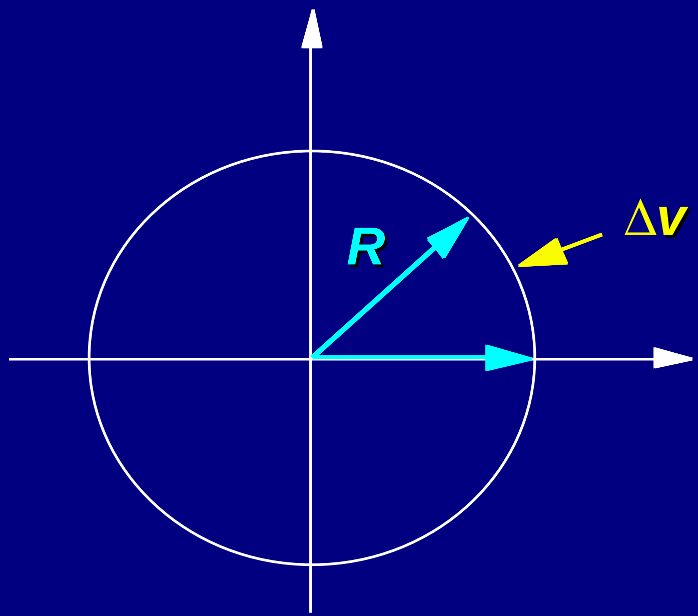


Move $\Delta \mathbf{v}$ to average time of the interval Δt.

Acceleration in UCM:

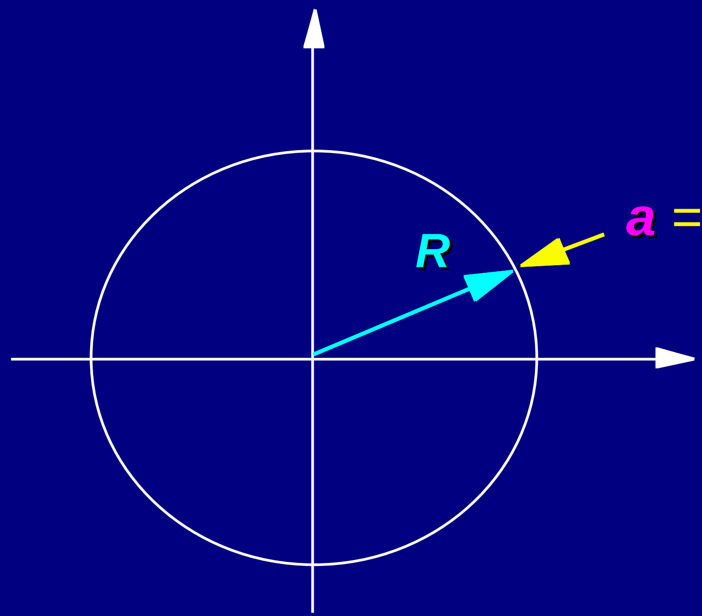
- Even though the *speed* is constant, velocity is ***not*** constant since the direction is changing.
  - Consider average acceleration in time Δt ⇨ $\Delta \boldsymbol{v} / \Delta t$



seems like $\Delta \boldsymbol{v}$ (hence $\Delta \boldsymbol{v}/\Delta t$) points toward the origin !

Acceleration in UCM:

- Even though the *speed* is constant, velocity is ***not*** constant since the direction is changing.
  - As we shrink Δt, $\Delta \mathbf{v} / \Delta t$ ⇨ $d\mathbf{v} / dt = \mathbf{a}$

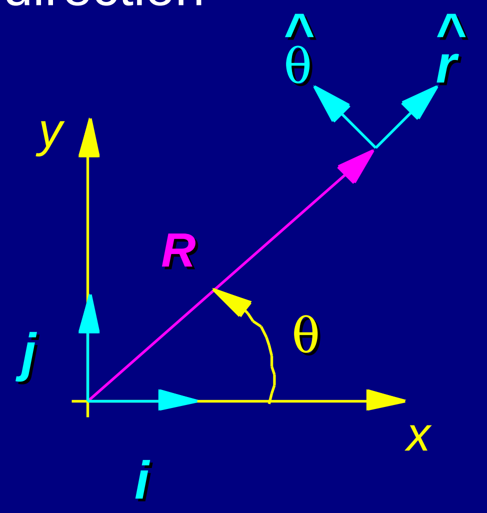


We see that ***a*** points in the - ***R*** direction.

Aside: Polar Unit Vectors

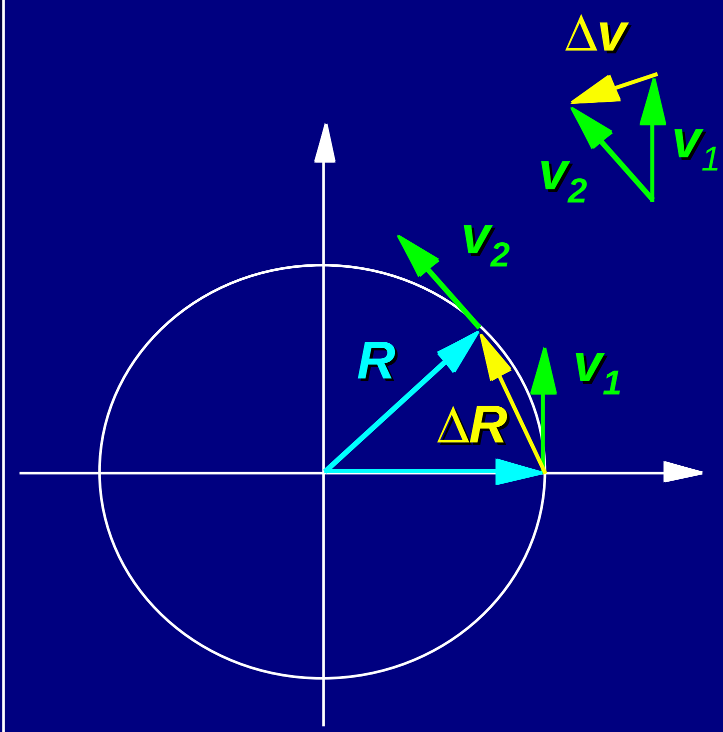
- We are familiar with the cartesian unit vetors: ***i*** ***j*** ***k***

- Now introduce
  "polar unit-vectors" $\hat{r}$ and $\hat{\theta}$:
  - $\hat{r}$ points in radial direction
  - $\hat{\theta}$ points in tangential (ccw) direction



Centripetal Acceleration

- Must be accelerating if direction is changing:
  - Centripetal Acceleration !!



Similar triangles: $\dfrac{\Delta v}{v} = \dfrac{\Delta R}{R}$

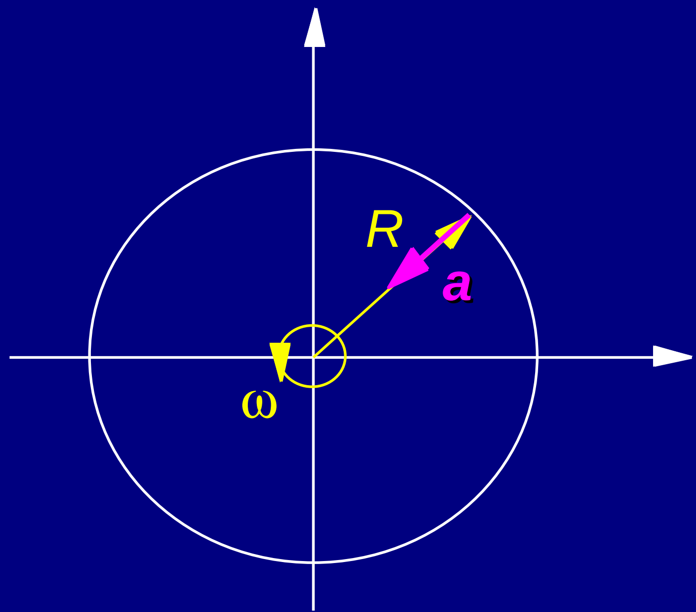
But $\Delta R = v\Delta t$ for small Δt

So: $\dfrac{\Delta v}{v} = \dfrac{v\Delta t}{R}$ ➡ $\dfrac{\Delta v}{\Delta t} = \dfrac{v^2}{R}$

$$\boxed{a = \frac{v^2}{R}}$$

Centripetal Acceleration

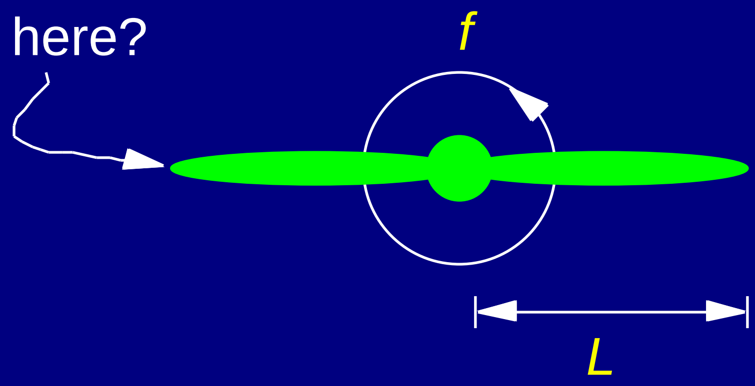
- UCM results in acceleration:
  - Magnitude: $a = v^2 / R$ (= $\omega^2 R$ since $v = R\omega$)
  - Direction: $-\hat{r}$ (toward center of circle)



Example: Propeller Tip

- The propeller on a stunt plane spins with frequency *f = 3500 rpm*. The length of each propeller blade is *L= 80cm*. What centripetal acceleration does a point at the tip of a propeller blade feel?

what is ***a*** here?

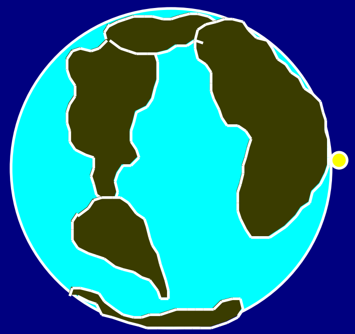


Example:

- First calculate the angular velocity of the propeller:
  - $$1\ rpm = 1\frac{rot}{min} x\ \frac{1}{60}\frac{min}{s} x\ 2\pi\frac{rad}{rot} = 0.105\frac{rad}{s} \equiv 0.105\ s^{-1}$$
  - so *3500 rpm* means $\omega = 367\ s^{-1}$
- Now calculate the acceleration.
  - $a = \omega^2 R = (367 s^{-1})^2 \times (0.8 m) = 1.1 \times 10^5\ m/s^2$
    $= 11{,}000\ g$
  - direction of ***a*** is toward propeller hub ($-\hat{r}$).

Example: Acceleration at Equator.

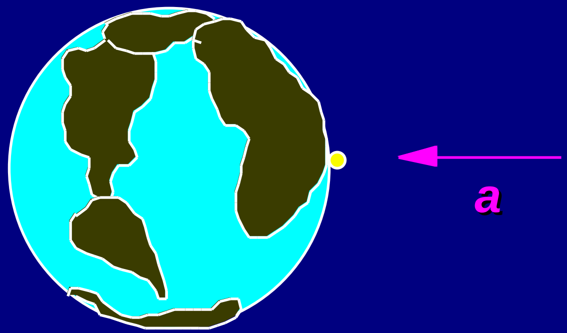
- What is the centripetal acceleration experienced by a person standing on the earth's equator, due to the earth's rotation.



- Recall that the radius of the earth is $R_e = 6.35 \times 10^6\ m$.

Example:

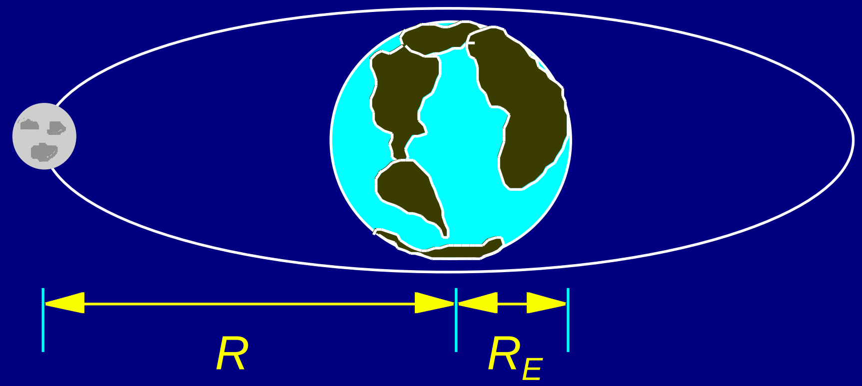
- First, figure out ω: $1\frac{rot}{day} x \frac{1}{86400}\frac{day}{s} x\ 2\pi\frac{rad}{rot} = 7.3x10^{-5}\ s^{-1}$



- Now calculate the acceleration.
  - $a = \omega^2 R$ = 0.034 m/s^2 = 0.3% of g
  - direction of ***a*** is toward center of earth ($-\hat{r}$).

Example: Newton & the Moon

- What is the acceleration of the Moon due to its motion around the earth?
- What we know (Newton knew this also):
  - T = 27.3 days = 2.36 x 10^6 s (period ~ 1 month)
  - R = 3.84 x 10^8 m (distance to moon)
  - R_E = 6.35 x 10^6 m (radius of earth)



Moon...

- Calculate angular frequency:

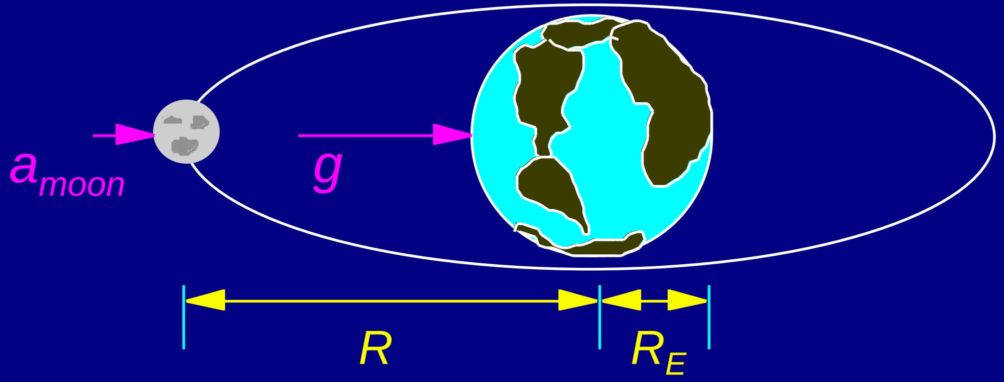
$$\frac{1}{27.3}\frac{rot}{day} x \frac{1}{86400}\frac{day}{s} x\ 2\pi\frac{rad}{rot} = 2.66x10^{-6}\ s^{-1}$$

- So $\omega = 2.66 \times 10^{-6}\ s^{-1}$.

- Now calculate the acceleration.
  - $a = \omega^2 R = 0.00272\ m/s^2 = .000278\ g$
  - direction of ***a*** is toward center of earth ($-\hat{r}$).

Moon...

- So we find that a_{moon} / g = .000278
- Newton noticed that R_E^2 / R^2 = .000273



- This inspired him to propose the
Universal Law of Gravitation: $F_{Mm} = GMm / R^2$
- What if our solar system was more complicated...
  - Would early scientists have been as successful??
  - Would science have evolved differently??

Recap for today:

- Reference frames and relative motion.
- Uniform Circular Motion
- Look at Textbook problems