MATH 137 – ASSIGNMENT #1

SOLUTIONS

page 85 #4 In order to solve $|x-1| - |x-3| \geq 5$ we have to unravel the absolute value signs. This forces us to consider x in the three intervals determined by 1 and 3.

If $x < 1$ we solve $-(x-1) - (-(x-3)) \geq 5$ which simplifies to $-2 \geq 5$. This is impossible. Thus no $x < 1$ works.

If $1 \leq x < 3$ we solve $x - 1 - (-(x-3)) \geq 5$ which simplifies to $2x - 4 \geq 5$, and hence $x \geq 9/2$

So the x's between 1 and 3 that solve the inequality have to be at least $4\frac{1}{2}$. There aren't any x's here either.

If $3 \leq x$ we solve $(x-1) - (x-3) \geq 5$ which comes down to $2 \geq 5$, which is completely impossible. So no x's bigger than 3 solve the inequality.

This inequality has <u>NO</u> solution at all.

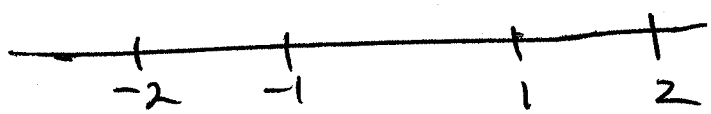
page 85 # 6 We have to sketch $y = |x^2-1| - |x^2-4|$

We can see that $|x^2-1| = \begin{cases} x^2-1 & \text{when } x^2 \geq 1 \\ 1-x^2 & \text{when } x^2 < 1. \end{cases}$

In other words $|x^2-1| = \begin{cases} x^2-1 & \text{when } x \leq -1 \\ 1-x^2 & \text{when } -1 < x < 1 \\ x^2-1 & \text{when } 1 < x \end{cases}$

Likewise $|x^2-4| = \begin{cases} x^2-4 & \text{when } x \leq -2 \\ 4-x^2 & \text{when } -2 < x < 2 \\ x^2-4 & \text{when } 2 \leq x. \end{cases}$

The critical cut-offs are $-2, -1, 1, 2$.

We look at 5 cases 

$x \leq -2 \Rightarrow y = (x^2-1) - (x^2-4) = 3$

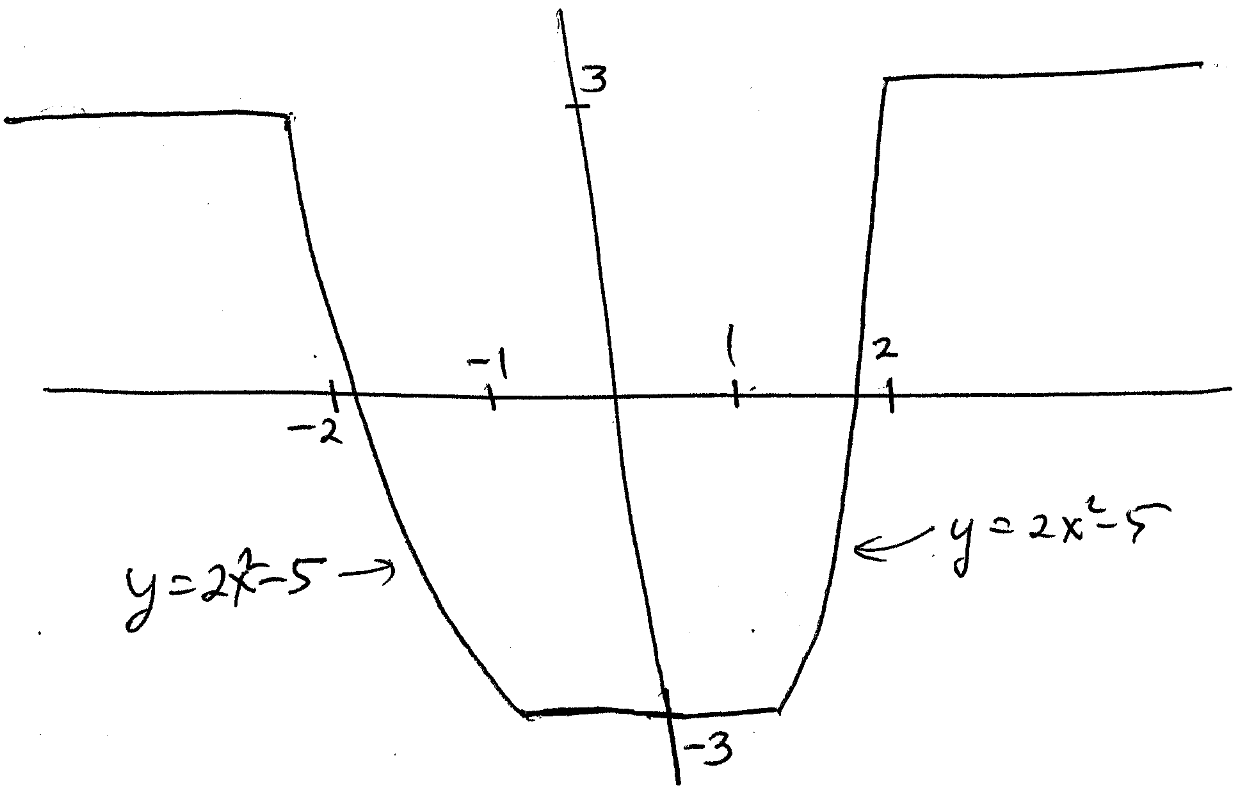
$-2 \leq x < -1 \Rightarrow y = (x^2-1) - (4-x^2) = 2x^2-5$

$-1 \leq x < 1 \Rightarrow y = (1-x^2) - (4-x^2) = -3$

$1 \leq x < 2 \Rightarrow y = (x^2-1) - (4-x^2) = 2x^2-5$

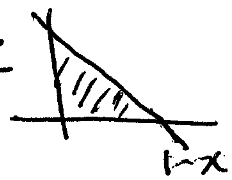
$2 \leq x \Rightarrow y = (x^2-1) - (x^2-4) = 3$

Here is a sketch of the graph

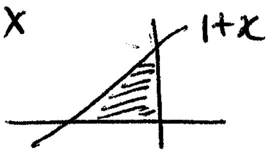


p. 85 # 9 To find those points (x,y) such that $|x|+|y|\le 1$ we consider 4 cases corresponding to the 4 quadrants

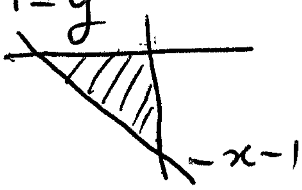
$0\le x, 0\le y$ give $x+y\le 1$ or $y\le 1-x$



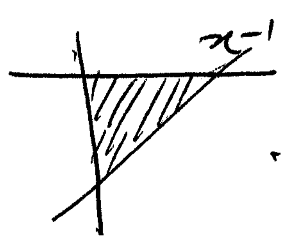
$x<0, 0\le y$ give $-x+y\le 1$ or $y\le 1+x$



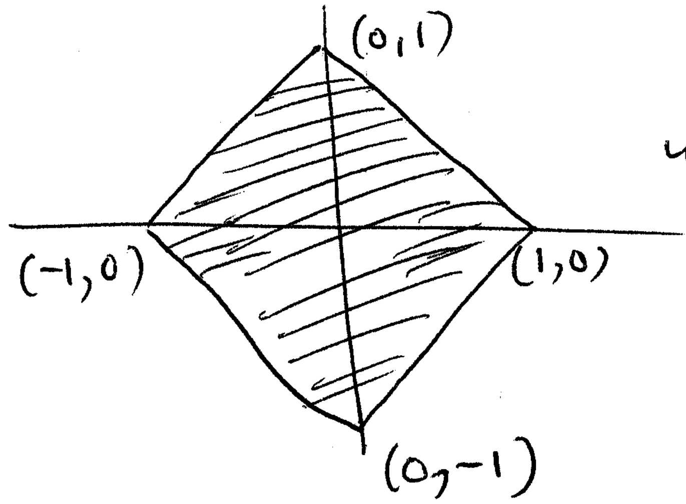
$x<0, y<0$ give $-x-y\le 1$ or $-x-1\le y$



$\le x, y<0$ give $x-y\le 1$ or $x-1\le y$



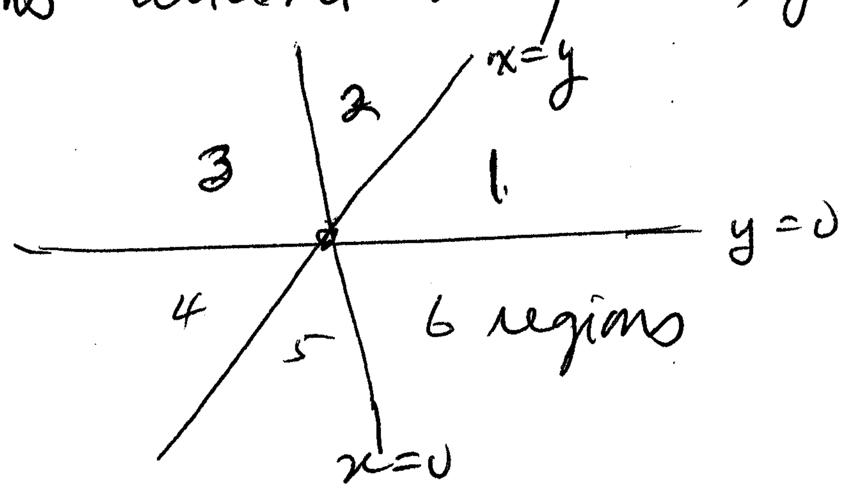
Put it all together to get a



with the edges included.

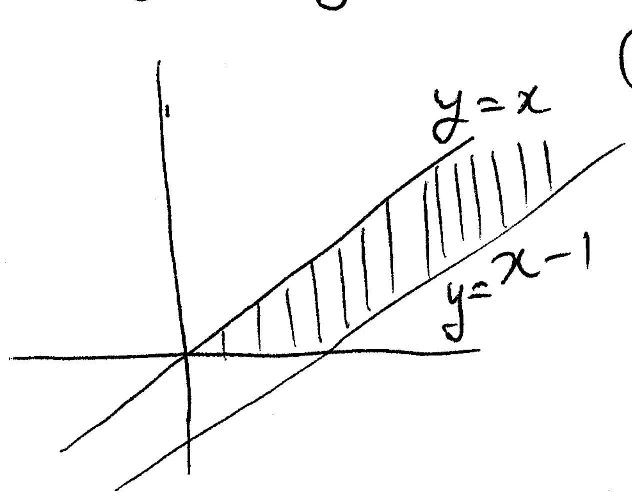
p. 85 #10

Again with patience we consider the 6 regions determined by $x=0$, $y=0$, $x=y$.



In Region 1 we get

$x\geq 0, y\geq 0$, $y\leq x$ & $|x-y|+|x|-|y|\leq 2$ becomes



$$(x-y)+x-y\leq 2$$
$$2x-2y\leq 2$$
$$x-y\leq 1$$
$$x-1\leq y$$

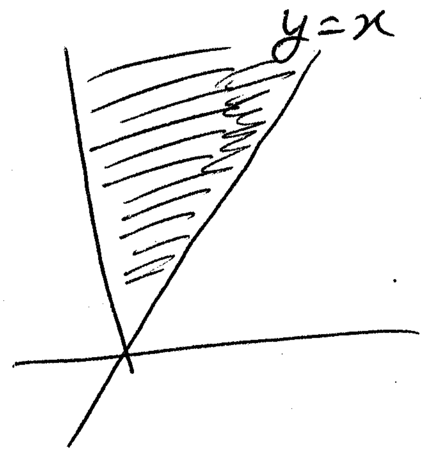
In region 2 we get.

$x \geq 0$, $y \geq 0$, $y \geq x$ & $|x-y| + |x| - |y| \leq 2$ becomes.

$(y-x) + x - y \leq 2$ which becomes

$0 \leq 2$.

This tells me all (x,y) in region 2 get picked up.



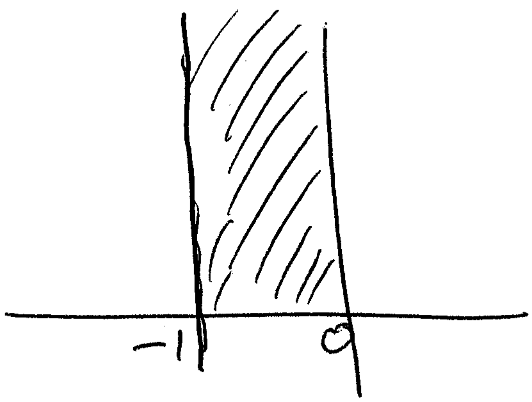
In region 3 we get

$x \leq 0$, $y \geq 0$, $y \geq x$ & $|x-y| + |x| - |y| \leq 2$ becomes

$y - x - x - y \leq 2$ or

$-2x \leq 2$ or

$x \geq -1$



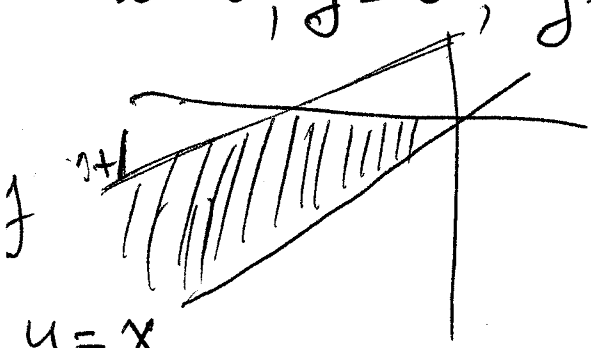
In region 4 we get

$x \leq 0$, $y \leq 0$, $y \geq x$. & $|x-y| + |x| - |y| \leq 2$ becomes

$(y-x) - x + y \leq 2$ which becomes

$2y - 2x \leq 2$

$y \leq x + 1$

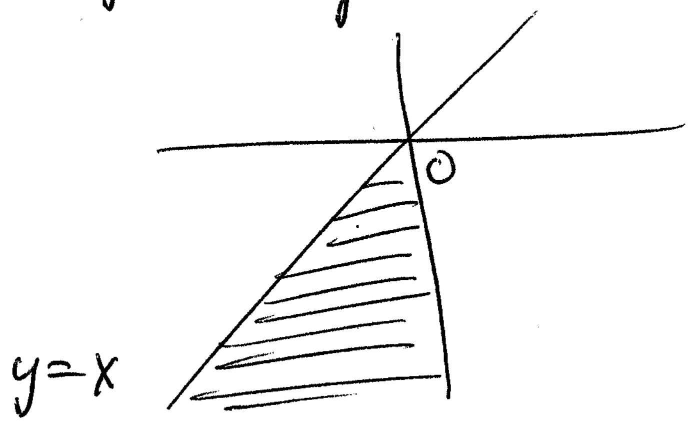


In region 5 we get.

$x \le 0,\ y \le 0,\ y \le x$ & $|x-y| + |x| - |y| \le 2$ becomes

$$(x-y) - x + y \le 2$$
$$0 \le 2.$$

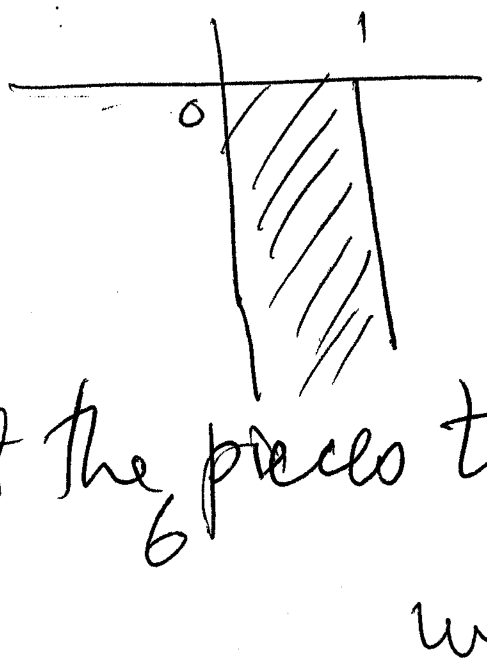
Here all (x,y) get picked up.



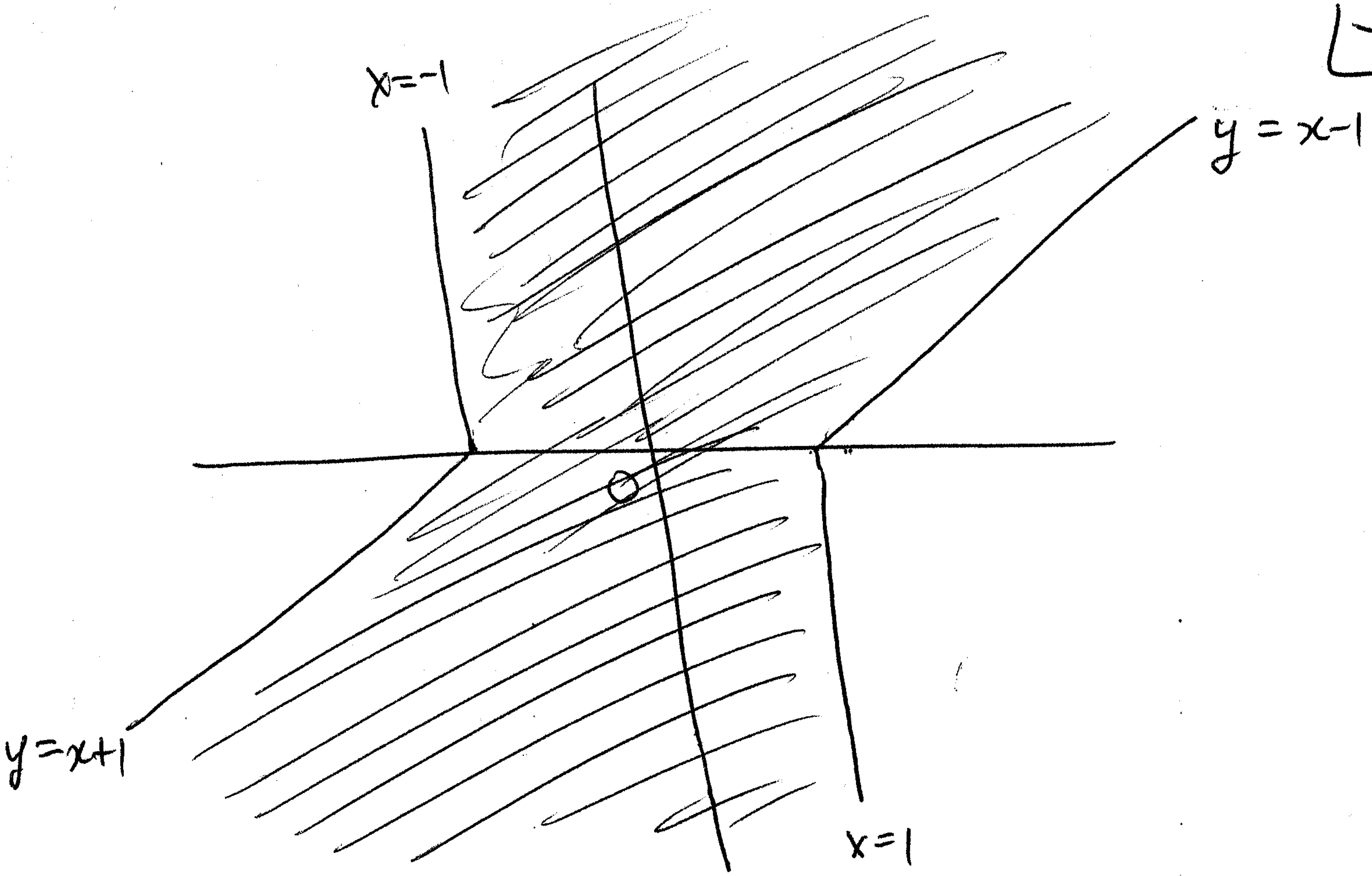
In region 6 we get.

$x \ge 0,\ y \le 0,\ y \le x$ & $|x-y| + |x| - |y| \le 2$ becomes

$$(x-y) + x - y \le 2$$
$$2x \le 2$$
$$x \le 1$$



Put the 6 pieces together and we get


x=-1
y=x-1
y=x+1
x=1

page 323 #2

We are asked to sketch the cubic

$$y = x^3 + 6x^2 + 9x$$

We see that $y = x(x+3)^2$.

Thus the x-intercepts are 0 and -3.

Let's do a sign inspection.

$x < -3$ gives $y < 0$.
$-3 < x < 0$ gives $y < 0$

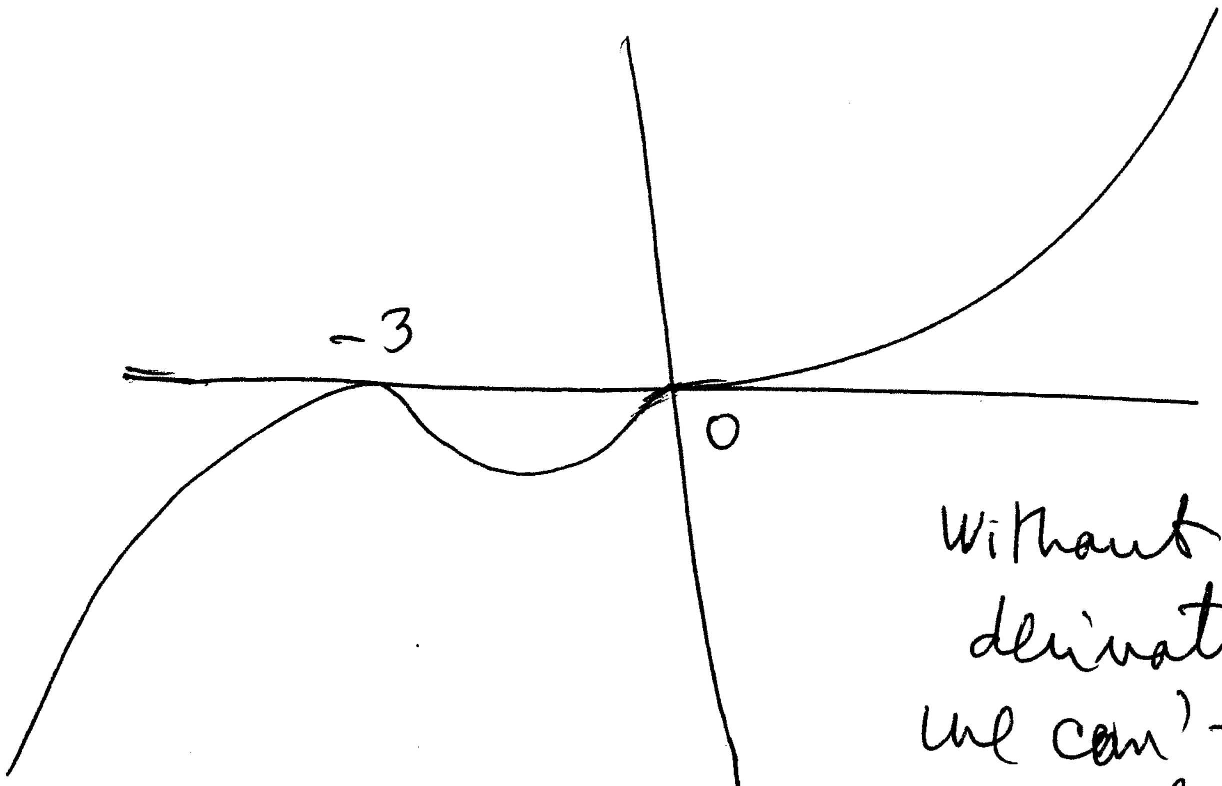
we could have done these two steps at once due to $(x+3)^2$

$0 < x$ gives $y > 0$.

What happens when x is big and > 0.
Well, y is big and > 0.

What happens when x is big and < 0.
Well, y is big and < 0.

Thus a likely picture is



Without derivatives we can't be sure where the minimum is at or where the inflections are.

However we already have a good idea of where these might be.

page 323 # 10 Sketch $y = \frac{x}{(x-1)^2}$.

We won't use derivatives. • domain is all $x \neq 1$

• x-intercepts at 0. (i.e. the function goes through origin)

• vertical asymptote at 1 i.e. when x is close to 1 the function blows up.

• Let's do a sign inspection of y for x between intercepts and asymptotes

$x < 0 \Rightarrow y < 0$

$0 < x < 1 \Rightarrow y > 0$

$1 < x \Rightarrow y > 0$.

• What happens when x is big & $x > 0$.

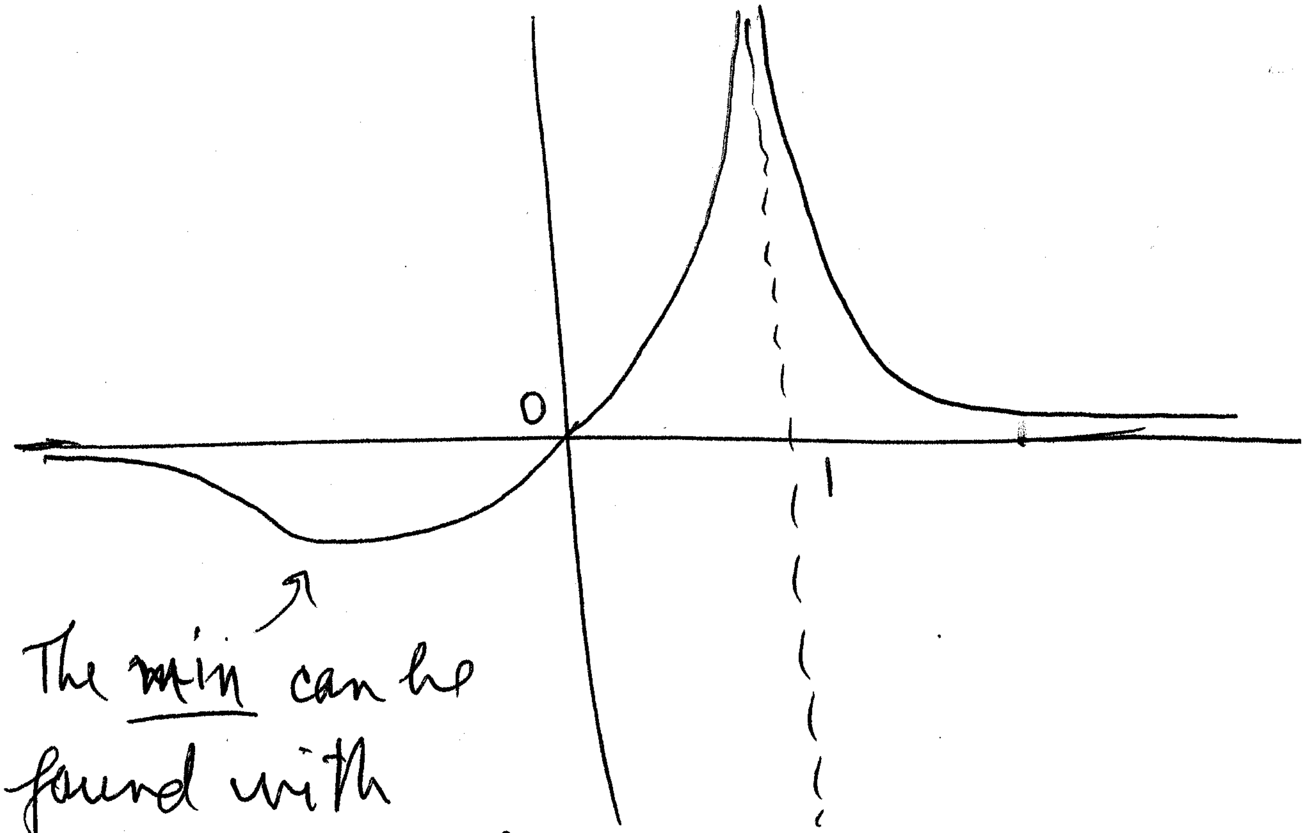
We have $y = \frac{1/x}{(1-1/x)^2} \approx 0$.

(i.e. $\lim_{x \to \infty} y = 0$). we divided top + bottom by x^2

• What happens when x is big & $x < 0$.

We get $y = \frac{1/x}{(1-1/x)^2} \approx 0$.

Thus the graph must look like.



The <u>min</u> can be found with derivatives, but we let that go for now.

<u>p. 323 #14</u>. Sketch $y = \frac{x^2}{x^2+9}$.

We see that domain is all x

~~Also~~ $\cancel{y = \frac{x^2+9}{x^2+9} + \frac{-9}{x^2+9} = 1 + \frac{9}{x^2+9}}$

~~this might come in handy~~.

- clearly $y \geq 0$ all the time
- and the function goes through the origin.
- y is an even function

- all we need to do now is worry about what happens when x is big.

Using long division or cleverness we see that $y = \frac{x^2}{x^2+9} = \frac{x^2+9}{x^2+9} - \frac{9}{x^2+9}$

$$= 1 - \frac{9}{x^2+9}.$$

$$= 1 - \frac{9/x^2}{1+9/x^2}.$$

When x is big & $x > 0$ we see that $y \approx 1$ & $y < 1$ because $\frac{9}{x^2+9} > 0$.

When x is big & $x < 0$ we see that $y \approx 1$ & $y < 1$ because $\frac{9}{x^2+9} > 0$.

The picture becomes.