Gravitational energy in Einstein's theory

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It is shown that in Einstein's theory of gravitation based on geometry of space-time and principle of equivalence of acceleration and gravitational field the *gravitational energy* of any object is its internal energy $E = mc^2$.

Derivation of results

To show that *gravitational energy* of any object is its internal energy $E = mc^2$, where m is inertial mass of the object and c is speed of light, it's enough to show that *gravitational force* $F := -\frac{dE}{dx} = mg$. In order to show it we differentiate internal energy with respect to distance x

$$\frac{dE}{dx} = \frac{d}{dx}(mc^2) = c^2 \frac{dm}{dx} + 2mc \frac{dc}{dx} \tag{1}$$

and see that since in static case $\frac{dm}{dx} = 0$ then an assumption

$$\frac{dc}{dx} = -\frac{g}{2c} \tag{2}$$

results in $\frac{dE}{dx} = -mg$ as required. Now we show that eq. (2) is a true assumption. For this purpose we consider bending of a light ray in gravitational field and in an accelerating rocket considering those effects identical on Einstein's principle of equivalence. In an accelerating rocket the light ray during its travel across the rocket is deflected by angle $\theta = \frac{gt}{c}$, where g is acceleration of the rocket and t is coordinate time that the light takes to travel accross the rocket. Since we know from the deflection of light in vicinity of masses that in gravitational field half of the angle of deflection comes from the curvature of space that doesn't cause any change in speed of light and the other half comes from the gravitational time dilation that causes change in coordinate speed of light between both sides of the light ray, then we conclude that half of θ is due to changing speed of light across the width of the light ray. So the angle due to changing speed of light is $\frac{\theta}{2}$ and $\frac{dc}{dx} = -\frac{\theta}{2t} = -\frac{g}{2c}$. Q.E.D.

It also follows from eq. (1) that in free fall (that applies to all isolated objects in the universe) $\frac{dE}{dx} = 0$. It demonstrates absence of *gravitational forces* in free fall (of course except the tidal forces of a static cases in the frames of the falling objects themselves). It prevents the existence of any *gravitational energy* outside the physical objects. Therefore the internal energy of objects is the only *gravitational energy* that there is in the universe.