EN1042-**Maple** ® Lab-02 Instructions

M Ghassempoory

January 23, 2006

1 Substitution

Use the **`subs`**, **`sort`**, **`collect`** and **`simplify`** commands to derive the following results

(a)

$$A = a\,x^3 + b\,x^2 + c\,x + d$$
$$x = X - \frac{b}{3\,a} \Rightarrow A = aX^3 + \left(c - \frac{b^2}{3a}\right) X - \frac{bc}{3a} + d + \frac{2b^3}{27a^2}$$

(b)

$$C = \frac{z-3}{z^2 - 3\,z + 2}$$
$$z = 2\,x + 6 \Rightarrow C = \frac{2\,x+3}{2\,(2\,x+5)\,(x+2)}$$

2 Identities

(a) Find A, B and C in the following identity

$$\frac{x^2 + 3\,x + 4}{(x+1)(x-2)(x+7)} \equiv \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+7}$$

(b) Find C_0, $C_1 \cdots C_4$ in the following identity

$$\frac{x^4}{(x+1)^3\,(x-1/2)\,(x+1/3)} \equiv \frac{C_0}{(x+1)} + \frac{C_1}{(x+1)^2} + \frac{C_2}{(x+1)^3} + \frac{C_3}{(x-1/2)} + \frac{C_4}{(x+1/3)}$$

(c) Verify cases (a) and (b) by using **`convert`** function with **`parfrac`** option.

3 Integration

(a) Evaluate the following integrals (some of them may not have an analytical answer):

$$\int_0^{x_0} x^5\,\mathbf{e}^{-x}\,\mathbf{d}\,x \qquad \int_0^{\infty} x^5\,\mathbf{e}^{-x}\,\mathbf{d}\,x \qquad \int_0^{\infty} x^{\frac{5}{3}}\,\mathbf{e}^{-x}\,\mathbf{d}\,x$$
$$\int_0^{x_0} sin(x)\,\sqrt{1-x^2}\,\mathbf{d}x \qquad \int_a^b \frac{\sin(x)}{\sqrt{x^3}}\,\mathbf{d}x \qquad \int \frac{1}{x^5+1}\,\mathbf{d}x$$

(b) Evaluate the following integrals.

$$\int \frac{1}{x^4+x^3+x^2+1}\,\mathbf{d}x \qquad \int_0^{10} \frac{1}{x^4+x^3+x^2+1}\,\mathbf{d}x \qquad \int_0^{a} \frac{1}{x^4+x^3+x^2+1}\,\mathbf{d}x$$

(c) The center of gravity of the half circle with radius R shown in figure 1-(a) is at point $(x_0, 0)$ where

$$x_0 = \frac{\int_{x=0}^{R} x\sqrt{R^2 - x^2}\,\mathbf{d}x}{\int_{x=0}^{R} \sqrt{R^2 - x^2}\mathbf{d}x}$$

find x_0.

(d) The center of gravity of the area under the half sine cycle shown in figure 1-(b) is at point (x_0, y_0) where

$$x_0 = \frac{1}{A}\int_{x=0}^{\pi} x\,\sin(x)\,\mathbf{d}x \qquad\qquad y_0 = \frac{1}{A}\int_{y=0}^{1} y\,[\pi - \arcsin(y)]\,\mathbf{d}y$$

where

$$A = \int_{x=0}^{\pi} \sin(x)\mathbf{d}x$$

find x_0 and y_0.

4 plotting

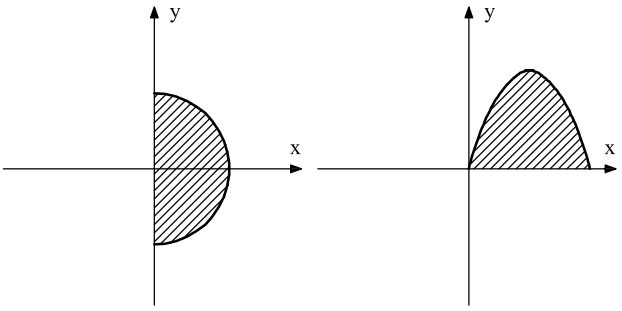
To plot an expression E for variable x in range $a \leq x \leq b$, you may use the **Maple** function **`plot`**:

```
plot(E,x=a..b)
```

Use this function to plot the following functions for $0 \leq x \leq \pi$

$sin(x)$ $\qquad$ $sin(x^2)$ $\qquad$ $\frac{sin(x)}{x+1}$ $\qquad$ $x\,sin(x)$

Look at the help section for **`plot`** command. Can you find a way of plotting all the given functions on a single plot?



(a) Half circle on (x, y) plane (b) Area under $\sin(x)$

Figure 1: Areas for center of gravity calculations.