Understanding Electromagnetics

Part I - Vector Analysis

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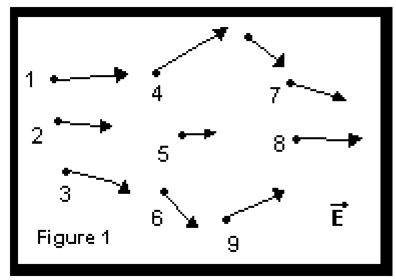
(7th Semester, ECE)

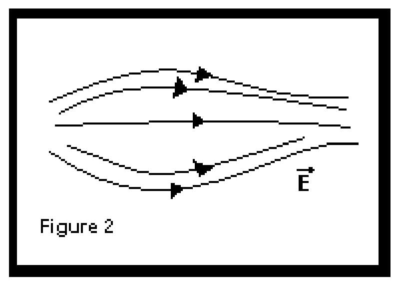
Chill! I'm not going to scare you with derivations or complex equations. Instead, I'm going to try and remove the misgivings in you regarding this, often under appreciated, subject.
Electromagnetics is, to be very precise, the study of how the world functions. Almost all the phenomena you observe around you have something to do with fields. Understanding Electromagnetics means understanding the real world. In this edition we will try to interpret the term **"Fields"** and also discuss the tool, which we use to study them – the vectors.

Fields

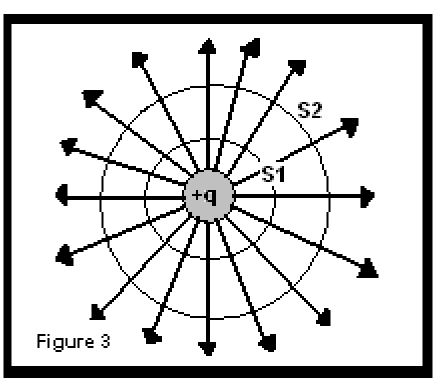


Lets first understand what the word Field means. Field is any quantity whose value varies in the space. Density, Temperature, Velocity of fluids etc are all fields. Here density and temperature have no direction, only magnitude, and hence are called scalar fields. Velocity has magnitude as well as direction and hence is called a vector field. At every point in space, thus, you can specify velocity in terms of the extent of its orientation towards the three axes **x**, **y** and **z**. The same is true for what we call the Electric and Magnetic Fields. Thus the Electric field can be described as the vector, **(**$E_i(x,y,z),E_j(x,y,z),E_k(x,y,z)$**)**, where subscripts i, j and k indicate the components along the three directions.

Lines of Forces



From the above description of fields we could graphically represent a vector field by marking vectors (pointed line segments) at as many points as possible in the space where the length indicates the magnitude and the arrow indicates the direction as shown in Figure 1. This was the conventional model used by early mathematicians. But physicists later devised a new, more useful way of representing vector fields- lines of force. In fact, most of our understanding of physics today is based on these lines of force.

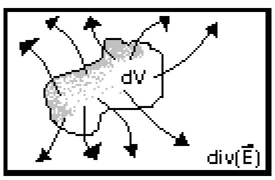


The convention goes like this: "the closer the lines are, the stronger the field and a line drawn tangential to the lines at any point gives the direction of the field at that point". So, instead of drawing longer vectors, we draw denser lines (See Figure 2). This representation gets along with intuition. Imagine a point charge +q (Figure 3), the field lines emanating from the charge are drawn. Now imagine two concentric spheres S1 and S2 around the charge. We can see that over the sphere S1, the density of lines is more than that over S2 as, being closer to the charge, its area is smaller; and hence by our convention, the field at any point on S1 is greater than S2. This, we can deduce from the common sense that as we move away from the charge the force experienced will get weaker and weaker (this "common sense" has a name – **Coulomb's Law**). Lines are good!

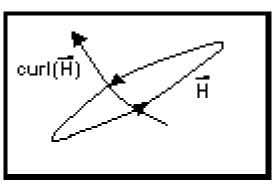
Vector Operators

You have come across the term **∇,** which translates to the vector, $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$. It is a vector operator, which by itself does not have any meaning but is an operator "hungry" to differentiate something. But when applied to fields, **it becomes a powerful machine for analysis**. Let us apply this operator to a scalar. Now, ∇V (read as 'gradient of V', where V is the electric potential, **a scalar field**) gives us a vector with components, the rate of change of V along **x**, **y** and **z** respectively. So, the vector ∇V will be tilted toward that axis along which the rate of change is maximum (for any vector is tilted toward **x** axis if the component along **x** is the greatest etc.). Thus the direction of gradient vector is the direction of the greatest change in a field.

When we write **E** = - ∇V, what we mean is that the electric field is pointing at the direction in which the potential (which is nothing but the potential energy per unit charge) DECREASES the fastest. This is the direction in which the force will act and hence, the field will point to. And that obviously will be the path taken by the test charge (unit positive charge). If this is confusing, imagine you are holding a stone at a height 'h' above the ground. You know that it will have a potential energy equaling the product of its mass, gravitational acceleration and the height 'h'. Now when you let go of the stone, it will fall straight downward and no other way. Because that is the only path which will enable the stone to loose its potential energy the fastest (remember- all objects hate energy). That is why the gravitational field always points toward the center of the Earth (on the assumption that all stars and other planets in the universe have disappeared!).



What happens if we apply the operator ∇ on a vector field? We can do this in two ways. First, we take a dot product of ∇ and a vector, say **E**. What we get is a scalar. It is called the "divergence of **E**". It is the summation of the rate of change of E_i along x direction, Ej along y direction and E_k along z direction. If we take **div(E)** in a small volume dV at the point P(x,y,z) in the space, it gives the increment (or decrement) in the magnitude of the vector when we move just out of the volume **(the sum of the differences between the components just outside and just inside the volume)**, or in other words, the "flux" diverging out of the given volume.



Another way to apply the operator to a vector field is cross product, **∇ X H**, where **H** is a vector field. This gives a vector. When you expand the expression, you will find that it is a vector, which sums up the circulation of the field **H** along an elemental loop and is normal to that loop. It is called **curl(H)**. For example, for a small loop in the x-y plane, we find that,

$$\lim_{\Delta x\,\Delta y \to 0} \frac{\oint \boldsymbol{H} \cdot d\boldsymbol{L}}{\Delta x\,\Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \nabla_x H_y - \nabla_y H_x = (\nabla \times \boldsymbol{H})_z$$

$$\text{where, } \nabla_x = \frac{\partial}{\partial x} \text{ and } \nabla_y = \frac{\partial}{\partial y}$$

Here, **H** = **(**H_x,H_y,H_z**)** and dL is the elemental distance along the loop with an area ΔxΔy around the point, P(x,y,z).

See the Lines

Whenever you see the expression **∇ . E**, you shouldn't think about the equations they represent but instead the first thing that should come to your mind must be field lines diverging out of a closed surface. Similarly, **∇ X H** should make you think about a loop of **H** circulating and in effect, giving a vector through the loop. Once you are able to see beyond the complexities of equations, Physics becomes beautiful. Mathematics is just the tool… what is more important is the physical interpretation.

In the next edition, we will look at the seven fundamental equations, which summarize the entire Classical Physics.