Understanding Electromagnetics

Part II – A God Named Maxwell

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(7th Semester, ECE)

Hi Folks! We will resume our journey from where we left off. As I had promised, I give you the seven fundamental equations of **Classical Physics**:

I. $\nabla \cdot B = 0$

II. $\nabla \cdot D = \rho$

III. $\nabla \times H = J + \frac{\partial D}{\partial t}$ … Maxwell's equations

IV. $\nabla \times E = -\frac{\partial B}{\partial t}$

V. $F = q(E + v \times B)$ … Force law

VI. $\frac{d}{dt}(p) = F,$ … Newton's Law of motion

where $p = \frac{mv}{\sqrt{1 - v^2/c^2}}$ (Einstein's modification)

VII. $F = -G \frac{m_1 m_2}{r^2} e_r$ … Gravitation

These equations, together with the four laws of Thermodynamics explain all the **Classical** concepts of the Universe. By "Classical", I mean the view that the early 19th century Physicists had about the way our world works- a view, which involved only those objects which are not **really very small** or **really very fast**. We will leave the equations **V**, **VI** and **VII** for the people studying mechanics. During the 1850s, a young professor in London named **James Clerk Maxwell** formulated the equations **I** to **IV** which have been named after him. These equations are fundamental to our discussion. In fact these four equations mathematically explain **every single law** or theorem you have read in electromagnetics. Don't believe it? Let me prove it…

B **– it's always a loop**

First consider Equation **(I)**. It says that the magnetic flux **B** (the magnetic flux lines per unit area) coming out of any closed surface is **always zero** (remember our definition of divergence?). This means that no matter what surface you consider anywhere in the universe, the magnetic flux going in must equal the magnetic flux going out. Hmmm… with a little contemplation you will realize that this will happen only if Magnetic Field lines occur always in **closed loops**. This is demonstrated in Fig.1. This is a fundamental concept of nature, which arises out of the fact that **there are no magnetic monopoles**- you cannot have an isolated North Pole or a South Pole as you can have an isolated positive or negative charge. The two poles always occur together in a magnet and no matter how many times you break it you will always end up with a dipole. In other words, the lines diverging out of a North Pole must always go through some South Pole and return back to where they started making a closed loop (See Fig.2). The reason for this is that **Magnetic Field is not an absolute field**. It is a secondary field formed as a result of a source Electric Field. We will discuss this in detail shortly.

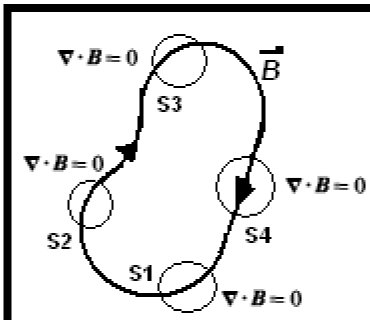


Figure 1: Divergence of $\vec{B}$, which is in the form of a loop,over four surfaces S1, S2, S3,S4

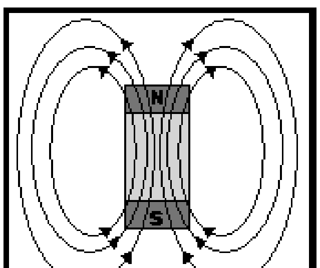


Figure 2

D **– it's a measure of charge**

Next consider **(II)**, it says that the electric flux D (the electric field lines per unit area) out of a closed volume is equal to the charge density inside that volume. This we all know is the **Gauss Divergence Theorem**. But let's analyze it for a special case shown in Fig.3. The charge **+q** is at the center of the sphere S of radius r, which is the surface under consideration. For this volume we can write, **ρ = q / (volume) = q /**

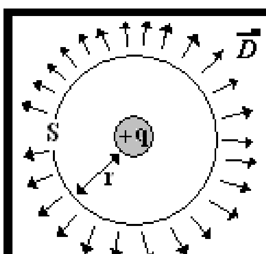


Figure 3

$(4/3)\pi r^3$. So, according to **(II)**, **$\nabla.D = q \;/\; (4/3)\pi r^3$**. By writing the divergence expression in spherical coordinates and taking advantage of the symmetry, we can simplify the above as **$D = q / 4\pi r^2$**, or **$E = q / 4\pi\varepsilon\, r^2$** (Never mind the derivations!). You know this is **Coulomb's Law**.

<u>Current gives me a Magnet</u>

Let us, for a while, forget the second term in the RHS of **(III)**. Then it says that around any wire with current density **J**, a loop of Magnetic Field (**$H = B/\mu$**) is formed (that's what curl means isn't it). We have read about this in school- **Ampere's Law**. Maxwell added the second term, **$\partial D/\partial t$** to this law. The reason is as follows: imagine this term wasn't there, then by taking divergence on both sides of equation, we find that **$\nabla.J = 0$** , since on the LHS, divergence of curl is zero. This goes to mean that there can never be any outflow of current from any volume- in other words, we can never have a current source! But this is not true- current sources do exist. If we pull out charges from volume, it contributes to current flowing out of that volume. So let's add **$\partial D/\partial t$** to the RHS and take divergence again. Now we find that **$\nabla.J = - \partial(\nabla.D) / \partial t$**. And by using **(I)** we can write, **$\nabla.J = - \partial\rho / \partial t$**. Which means that rate of decrease of charge density inside any volume is the current density outflow- meaning, charge disappearing inside must appear outside- point form of **Law of Conservation of Charge**! The diagrammatic representation of **(III)** is shown if Fig. 4.

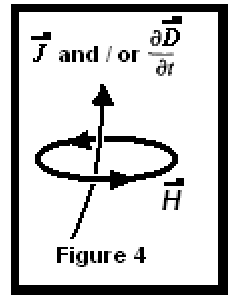


Figure 4

<u>I can generate Electricity!</u>

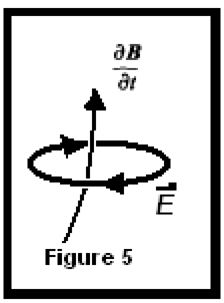


Figure 5

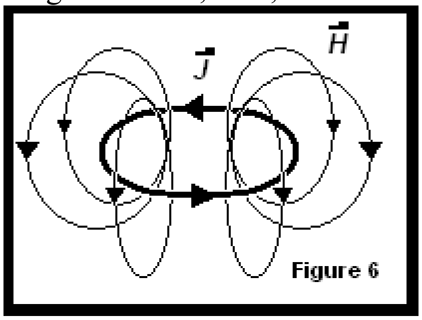
Finally, let us look at **(IV)**. It says that if Magnetic Field is varying with time, a loop of Electric Field is formed around it, in a sense opposite to case we saw above… ring any bells? … **Faraday's Law of Electromagnetic Induction**. Of course, in school, we studied a special case of this equation where a conductor loop is placed in the vicinity of a time-varying Magnetic Field and thus an **emf** was induced across it, which forms the basis of Power Generation. In the absence of a **conductor loop**, an **electric field loop** is generated in the space. Its representation is shown in Fig. 5. Note that the sense of rotation of **E** is opposite to that of **H** in Fig.4- the result of the negative sign in **(III)**. We see from the equations **(II)**, **(III)** and **(IV)** that while Electric Field can be generated either by charges or by varying Magnetic Field, Magnetic Field is generated only by time varying Electric Field (and current, which is also a function of Electric Field, **$J = \sigma E$**). So, as said before, **E** (or **J**) is the cause, **H** is the effect. Magnetic Field cannot exist by it self in Nature. Even the Magnetism in Permanent Magnets is due to the tiny current loops formed by the electrons inside Iron atoms.

<u>Maxwell is God</u>

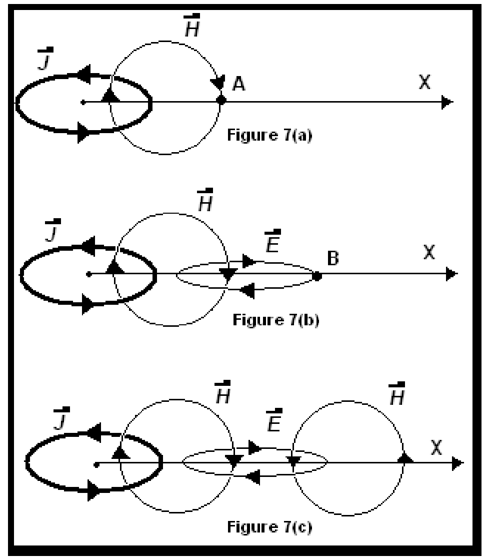
So we see that Maxwell has given a precise and yet simple mathematical representation of all the Laws of Electrodynamics. These four equations are completely exhaustive- if you know Maxwell's equations, you needn't know anything else in Electromagnetics. If you are given any scenario with certain charge and current distributions and asked to find the field patterns that arise, all you do is substitute the given conditions in the Maxwell's equations and solve them. It works for absolutely any situation. From resistors to transformers, from motors to generators, from computers to communication, these equations are omnipresent in our lives. From a long view of the history of mankind- seen from, say, ten thousand years from now- there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War and the Great Indian Mutiny of 1857 will pale into provincial insignificance in comparison with this important scientific breakthrough of the same decade.

Now What's This???

As a finishing note, consider a circuit powered by some source, with a constant current density **J = J$_0$** flowing in it (Fig. 6). From **(III)** a loop of Magnetic Field proportional to **J$_0$** will be formed, because of this current as shown, around different elemental lengths on the loop. This Magnetic Field, thus, will also be constant w.r.t. time. Substitute this Magnetic field in **(IV)** and we get **E = 0**. The story ends here (This Magnetic Field, being constant with time cannot produce any **E**).



Now what if the source was generating not a constant current but a current, which is a function of time- to be precise, a function that never "dies" no matter how many times you differentiate it (Sine, cosine, exponential etc.). So, if **J = J$_0$ sin (t)**, then the Magnetic Field **H** also will be a function of **sin(t)** (Fig. 7 (a), only the fields along **x** direction is shown although the argument can be extended to all directions). Now, at the point **A** there is a time varying Magnetic Field facing downward and hence, from **(IV)** an Electric Field- a function of **cos (t)**- is formed around it (Fig. 7(b)). Now, at point **B** there is a time varying Electric Field (coming out of the plane of this paper) and hence, from **(III)** a Magnetic Field, which is a function of **sin (t)** is formed around it (Fig. 7 (c)). We can keep doing this forever, as the sinusoidal functions are immune to differentiation. It's weird isn't it? We are witnessing a situation wherein, the fields no longer need a source and are self-sustaining themselves (**E** producing **H** and **H** in turn, producing **E** and so on…) and besides, are moving away from the source.



In the next issue we will see more about this "strange" phenomenon, which probably started the moment God said **"Let there be Light"**!!!