AP Calculus "Cheat Sheet"

Limits

Non-removable discontinuity = limit fails if:

1. Approaches different L and R values
2. Oscillates
3. Approaches $\pm$ infinity

Removable discontinuity is replaceable

If $\lim_{x\to c}\frac{f}{g}=\frac{\#}{0}$ then c = V.A.

$°n = °d$ H.A = ratio

$°n < °d$ H.A = 0 (1/x)

$°n > °d$ H.A = ∞ (x^2)

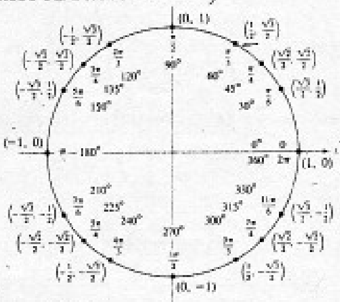
L'Hopital's Rule: If $\lim_{x\to c}\frac{f}{g}=\frac{0}{0}$ use $\frac{f'}{g'}$ or find another equation (cancel/rationalize)

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\cos x-1}{x}=0$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

$$\lim_{\Delta x\to 0}\left(\frac{f(x+\Delta x)-f(x)}{\Delta x}\right)=f'(x)$$



Derivatives

Power Rule: $\frac{d}{dx}(x^n)=nx^{n-1}$

Product Rule: $\frac{d}{dx}(fg)=f'g+fg'$

Quotient Rule: $\frac{d}{dx}\left(\frac{f}{g}\right)=\frac{f'g-fg'}{g^2}$

Chain Rule: $\frac{d}{dx}(f(u))=f'(u)\cdot u'$

$$\frac{d}{dx}(\sin u)=\cos u\cdot u'$$

$$\frac{d}{dx}(\cos u)=-\sin u\cdot u'$$

$$\frac{d}{dx}(\sec u)=\sec u\tan u\cdot u'$$

$$\frac{d}{dx}(\csc u)=-\csc u\cot u\cdot u'$$

$$\frac{d}{dx}(\tan u)=\sec^2 u\cdot u'$$

$$\frac{d}{dx}(\cot u)=-\csc^2 u\cdot u'$$

$$\frac{d}{dx}(\arcsin u)=\frac{1}{\sqrt{1-u^2}}\cdot u'=\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}(\arccos u)=-\frac{1}{\sqrt{1-u^2}}\cdot u'$$

$$\frac{d}{dx}(arc\sec u)=\frac{1}{|u|\sqrt{u^2-1}}\cdot u'$$

$$\frac{d}{dx}(arc\csc u)=-\frac{1}{|u|\sqrt{u^2-1}}\cdot u'$$

$$\frac{d}{dx}(\arctan\cdot u)=\frac{1}{1+u^2}\cdot u'$$

$$\frac{d}{dx}(arc\cot u)=-\frac{1}{1+u^2}\cdot u'$$

$$\frac{d}{dx}(e^x)=e^x \qquad \frac{d}{dx}(e^u)=e^u\cdot u'$$

$$\frac{d}{dx}(\ln x)=\frac{1}{x} \qquad \frac{d}{dx}(\ln|u|)=\frac{1}{u}\cdot u'$$

$$\frac{d}{dx}(a^x)=a^x\ln a \qquad \frac{d}{dx}(a^u)=a^u\ln a\cdot u'$$

$$\frac{d}{dx}(\log_a x)=\frac{1}{x\ln a} \qquad \frac{d}{dx}(\log_a u)=\frac{1}{u\ln a}\cdot u'$$

$$\left(f^{-1}\right)x=\frac{1}{f'\left(f^{-1}(x)\right)}$$

Derivative Applications

$f'(x)$ is the slope of the tangent line to a point on the curve. Also instantaneous v.

Critical Point(s): $f'(x)=0$

Increasing: $f'(x)>0$

Decreasing: $f'(x)>0$

Point(s) of Inflection: $f''(x)=0$

Concave Up: $f''(x)>0$ ☺

Concave Down: $f''(x)<0$ ☹

MVT: $f'(c)=\frac{\Delta y}{\Delta x}=\frac{f(b)-f(a)}{b-a}$

$s(t)=-\frac{1}{2}gt^2+v_it+h_i$ $g=9.8$ m/s^2 32 ft/s^2

$s(t)$ is the position function

$v(t)=s'(t)$ is the velocity function

$a(t)=v'(t)=s''(t)$ is the acceleration

* Differentiability implies continuity but not vice versa (sharp corners, vertical tangents)
* Related Rates: what's changing with respect to what?
* Optimization: Find the min/max of ___

Integration

Riemann Sums: $\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$,

$\Delta x = \frac{b-a}{n}$, $c_i = a + i\Delta x$, n many rectangles

Power Rule for Integrals: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int \cos u \cdot u' dx = \sin u + C$

$\int \sin u \cdot u' dx = -\cos u + C$

$\int \sec u \tan u \cdot u' dx = \sec u + C$

$\int \csc u \cot u \cdot u' dx = -\csc u + C$

$\int \sec^2 u \cdot u' dx = \tan u + C$

$\int \csc^2 u \cdot u' dx = -\cot u + C$

$\int \tan u \cdot u' dx = -\ln|\cos u| + C$

$\int \cot u \cdot u' dx = \ln|\sin u| + C$

$\int \sec u \cdot u' dx = \ln|\sec u + \tan u| + C$

$\int \csc u \cdot u' dx = -\ln|\csc u + \cot u| + C$

$\int \frac{u'}{\sqrt{a^2 - u^2}} dx = \arcsin\frac{u}{a} + C$

$\int \frac{u'}{a^2 + u^2} dx = \frac{1}{a}\arctan\frac{u}{a} + C$

$\int \frac{u'}{u\sqrt{u^2 - a^2}} dx = \frac{1}{a} arc\sec\frac{|u|}{a} + C$

$\int e^x dx = e^x + C$ $\int e^u \cdot u' dx = e^u + C$

$\int \frac{1}{x} dx = \ln|x| + C$ $\int \frac{1}{u} \cdot u' dx = \ln|u| + C$

$\int a^x \ln a dx = a^x + C$ $\int a^u \ln a \cdot u' dx = a^u + C$

$\int \frac{1}{u \ln a} \cdot u' dx = \log_a u + C$

Fundamental Theorem:

Part 1 $\int_a^b f(x)dx = F(b) - F(a)$

Part 2:

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

$$\frac{d}{dx}\left(\int_a^u f(t)dt\right) = f(u)\cdot u'$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Chain Rule: $u = g(x)$ and $u' = g'(x)dx$,

$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C$

$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

If f is even, $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

If f is odd, $\int_{-a}^{a} f(x)dx = 0$

Integration Applications

MVTI: $\int_a^b f(x)dx = f(c)(b-a)$

Ave value $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

Area between $f(x)$ and $g(x)$ if $g(x) \le f(x)$

$\int_a^b [f(x) - g(x)]dx$

Disk Method: $\pi\int_a^b (r(x))^2 dx$

Washer Method: $\pi\int_a^b (R(x))^2 - (r(x))^2 dx$

Shell Method: $2\pi\int_a^b d(x)h(x)dx$

Power Rule for Integrals: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Exponential growth/decay: $\frac{dy}{dt} ky; y = Ne^{rt}$

Area under a curve = total distance traveled.

Trapezoidal rule: $A = \int_a^b f(x)dx = \frac{b-a}{2n}\left(f(a) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(b)\right)$

Logarithmic Properties

Domain: $(0, \infty)$ Range $(-\infty, \infty)$

Continuous, increasing, 1:1

$\ln(ab) = \ln a + \ln b$

$\ln(a^n) = n \ln a$

$a^x = e^{(\ln a)x}$

Miscellaneous

y-symmetry: $f(-x) = f(x)$ (even)

x-symmetry: $-f(x) = f(x)$

origin-symmetry: $f(-x) = -f(x)$ (odd)

$y = f(x \pm c)$ shifts left (+) and right (–)

$y = f(x) \pm c$ shifts up (+) and down (–)

$y = -f(x)$ reflects about the x-axis

$y = f(-x)$ reflects about the y-axis

Inverse iff one-to-one (1:1), passes horizontal and vertical line test.

SOHCAHTOA $\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$