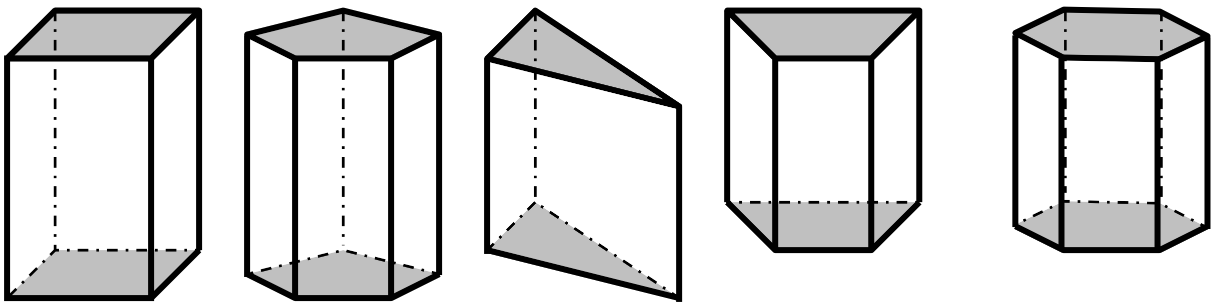
Sect 9.1 - Prism

Objective 1   Understanding Prism, Lateral Surface Area and Surface Area.

A **Prism** is a solid figure with congruent bases that are parallel and with sides that are parallelograms. The perpendicular distance between the bases is called the height or altitude. The sides of the prism that are not the bases are called the lateral sides. In this course, we will only consider prisms where the lateral sides are perpendicular to the bases. We call these types of prisms "right prisms." In each prism below, the congruent bases that are parallel are shaded in gray and the sides are in white.



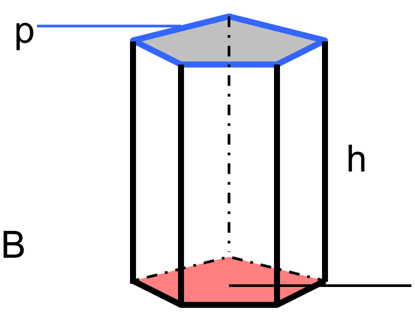
Rectangular Prism	Pentagonal Prism	Triangular Prism	Trapezoidal Prism	Hexagonal Prism

Properties of Right Prisms:

Let p = the perimeter of the base
B = the area of the base
h = the height of the Prism

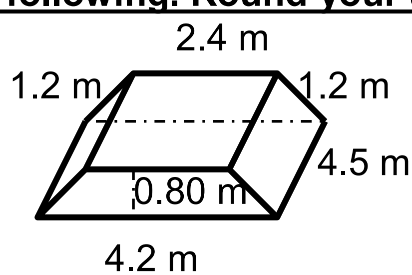
Then

1) Lateral surface area: $L = ph$
2) Total surface area: $SA = L + 2B$
3) Volume: $V = Bh$

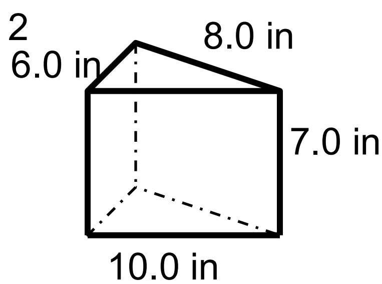


Find a) the lateral surface area, b) the surface area, and c) the volume of the following. Round your answers to three significant digits:

Ex. 1

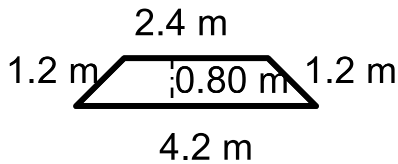


Ex. 2



Solution:

This figure is a prism turned on its side. The base is a trapezoid. First, we need to find the perimeter and area of the base:



$p = 1.2 + 2.4 + 1.2 + 4.2$
$= 9.0$ m

$B = \frac{1}{2}(b_1 + b_2)h$

where $b_1 = 4.2$, $b_2 = 2.4$, $h = 0.8$

$B = \frac{1}{2}(4.2 + 2.4)(0.8)$
$= \frac{1}{2}(6.6)(0.8) = 2.64 \text{ m}^2$

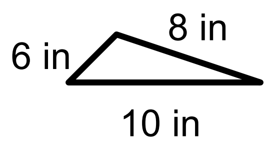
a) $L = ph = 9(4.5) = 40.5 \text{ m}^2$

b) $SA = L + 2B = 40.5 + 2(2.64)$
$= 40.5 + 5.28 = 45.78 \text{ m}^2$
$\approx 45.8 \text{ m}^2$

c) $V = Bh = 2.64(4.5) = 11.88 \text{ m}^3$
$\approx 11.9 \text{ m}^3$

Solution:

The base of the prism is a triangle. First, we need to find the perimeter and the area of the base:



$p = 6 + 8 + 10$
$= 24$ in

$B = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = P/2 = 24/2 = 12$

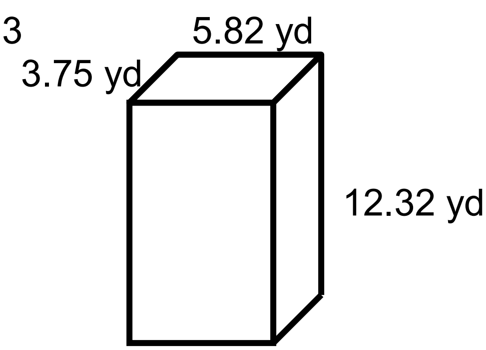
$B = \sqrt{12(12-6)(12-8)(12-10)}$
$= \sqrt{12(6)(4)(2)}$
$= \sqrt{576} = 24 \text{ in}^2$

a) $L = ph = 24(7) = 168 \text{ in}^2$

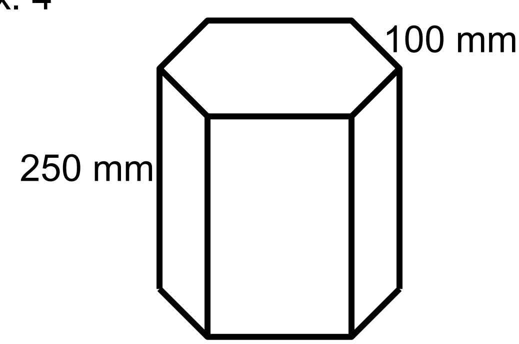
b) $SA = L + 2B = 168 + 2(24)$
$= 168 + 48 = 216 \text{ in}^2$

c) $V = Bh = 24(7) = 168 \text{ in}^3$

Ex. 3



Ex. 4



Assume the hexagon is regular.

Solution:

This base of the prism is a rectangle. First, we need to find the perimeter and the perimeter and area of the base:

5.82 yd

3.75 yd

$p = 2(5.82) + 2(3.75) = 11.64 + 7.5$
$= 19.14$ yd

$B = Lw$

$B = 5.82(3.75) = 21.825$ yd^2

a) $L = ph = 19.14(12.32)$
$= 235.8026 \approx 236$ yd^2

b) $SA = L + 2B$
$= 235.8026 + 2(21.825)$
$= 235.8026 + 43.65$
$= 279.4526 \approx 279$ yd^2

c) $V = Bh = 21.825(12.32)$
$= 268.884 \approx 269$ yd^3

Solution:

The base of the prism is a hexagon. First, we need to find the perimeter and the area of the base:

100 mm

$p = 6(100)$
$= 600$ mm

$$B = \frac{3a^2\sqrt{3}}{2}$$

$$B = \frac{3(100)^2\sqrt{3}}{2} = 15000\sqrt{3}$$
$= 25980.762\ldots$ mm^2

a) $L = ph = 600(250)$
$= 150000$ mm^2

b) $SA = L + 2B$
$= 150000 + 2(25980.7\ldots)$
$= 150000 + 51961.5\ldots$
$= 201961.5\ldots$
$\approx 202{,}000$ mm^2

c) $V = Bh = 25980.7\ldots(250)$
$= 6495190.52\ldots$
$\approx 6{,}500{,}000$ mm^3

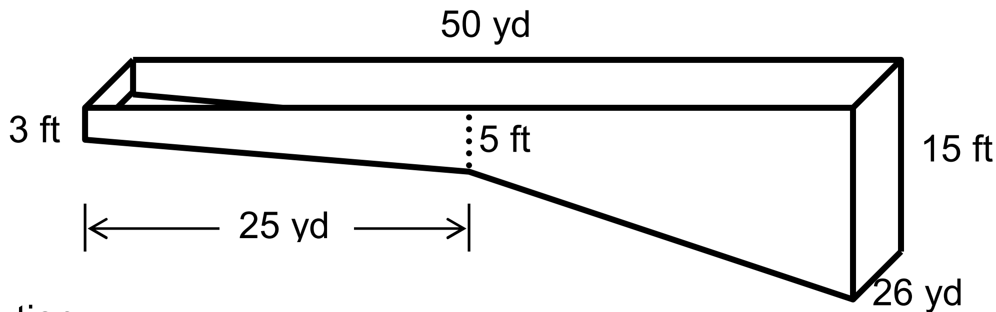
Objective 2: Applications involving the volume of prisms.

In many applications, after finding the volume, we will need to convert the units to the appropriate set of units to answer the question. Here are a list of common conversions we will need.

1 ft^3 = 1728 in^3 1 gal = 231 in^3 1 bu ≈ 2150.42 in^3 ≈ 1.24446 ft^3 1 pt = 28.875 in^3 1 yd^3 = 27 ft^3 = 46,656 in^3 1 fl oz ≈ 1.805 in^3 1 ft^3 ≈ 7.48052 gal	1 in^3 ≈ 16.3871 cm^3 1 ft^3 ≈ 0.0283168 m^3 1 ft^3 ≈ 28.3168 L 1 yd^3 ≈ 0.7646 m^3 1 fl oz ≈ 29.574 cm^3 1 qt ≈ 0.94635 L 1 gal ≈ 3.7854 L

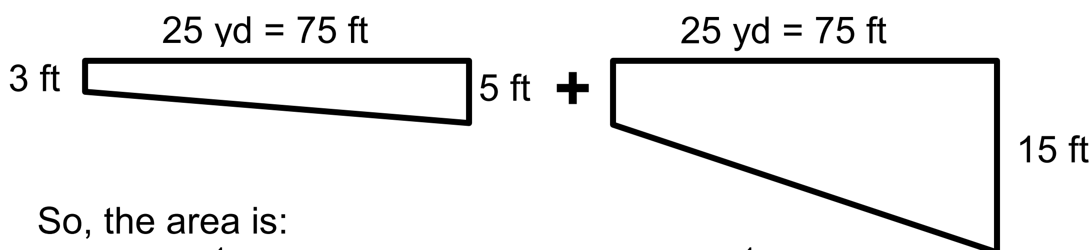
<u>Solve the following. Round to three significant digits:</u>

Ex. 5 How many gallons of water will be needed to fill the swimming pool pictured below?



<u>Solution:</u>

We can think of this as a prism turned on its side. We will first calculate the area of the base. The base is composed of two trapezoids turned on their sides. The height of the bigger trapezoid is 50 yd – 25 yd = 25 yd. But 25 yd = 25 yd (3ft/yd) = 75 feet:



So, the area is:

$$B = \frac{1}{2}(b_1 + b_2)h \quad + \quad \frac{1}{2}(b_1 + b_2)h$$

$$= \frac{1}{2}((3) + (5))(75) \quad + \quad \frac{1}{2}((5) + (15))(75)$$

$$= \frac{1}{2}(8)(75) + \frac{1}{2}(20)(75) = 300 + 750 = 1050 \text{ ft}^2$$

Since 26 yd = 26 yd (3 ft/yd) = 78 ft is the height of the prism, then the volume is

$$V = Bh = (1050)(78) = 81900 \text{ ft}^3.$$

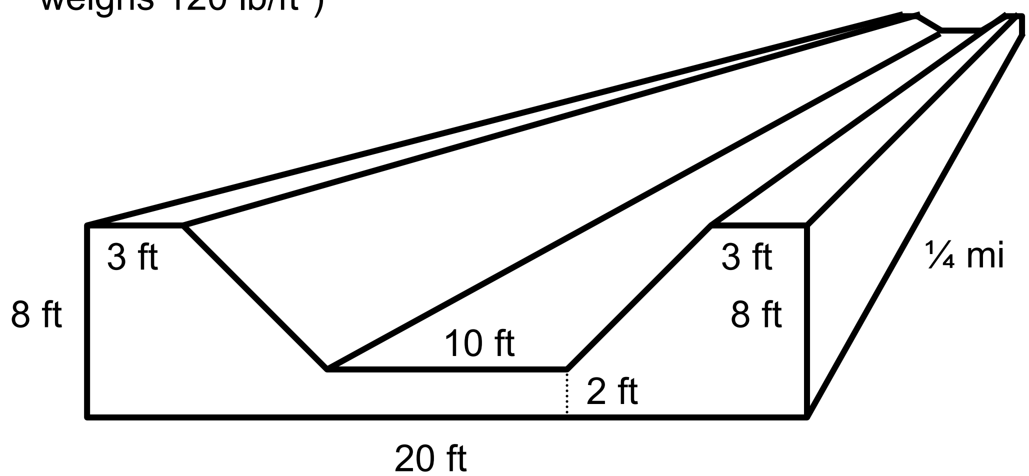
But, 1 ft^3 = 7.48052 gallons, so

$$81900 \text{ ft}^3 = \frac{81900 \text{ ft}^3}{1} \bullet \frac{7.48052 \text{ gal}}{1 \text{ft}^3} = 612654.588 \text{ gal}$$

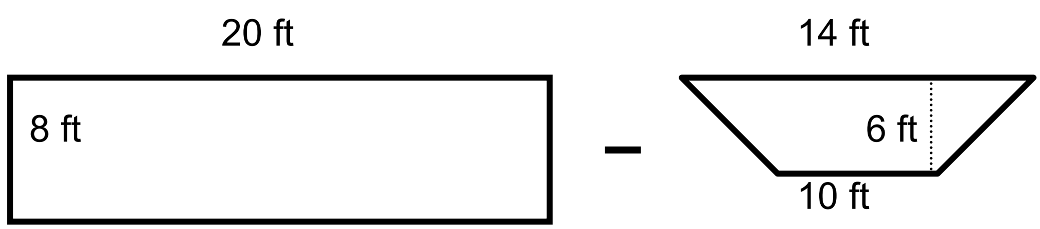
$$\approx 613{,}000 \text{ gal.}$$

Hence, 613,000 gallons of water will be needed to fill up the swimming pool.

Ex. 6 How many tons of concrete must be poured to create the drainage canal illustrated below? (assume that the concrete weighs 120 lb/ft^3)



<u>Solution:</u>
We can think of this as a prism turned on its side. We will first calculate the area of the base. The base is a rectangle that has had a trapezoid cut out of it, so its area is the area of a rectangle minus the area of a trapezoid. The longer base of the trapezoid is 20 – 3 – 3 = 14 ft and the height of the trapezoid is 8 – 2 = 6 ft



$$B = Lw - \frac{1}{2}(b_1 + b_2)h$$

$$= (20)(8) - \frac{1}{2}((10) + (14))(6)$$

$$= 160 - \frac{1}{2}(24)(6) = 160 - 72 = 88 \text{ ft}^2$$

The height of the prism is $\frac{1}{4}$ mi = $\left(\frac{1}{4}\text{ mi}\right)$ (5280 ft/ mi) = 1320 ft.

So, the volume is:

$$V = Bh = 88(1320) = 116{,}160 \text{ ft}^3$$

The weight is: (116,160 ft^3)(120 lb/ft^3) = 13,939,200 pounds.
Converting into tons, we get: (13,939,200 lb) (1 ton/2000 lb) = 6969.6 tons ≈ 6,970 tons
Hence, 6,970 tons of concrete will need to be poured.