Sect 8.4 – Circles

Objective 1: The Circumference and Area of a Circle.

Recall the following from the Geometry Review:

Circumference of a Circle

If d is the diameter of a circle and r is the radius of the circle, then the circumference, C, of the circle is

$C = \pi d$ or $C = 2\pi r$ where $\pi \approx 3.1415926535 \ldots$

Area of a Circle

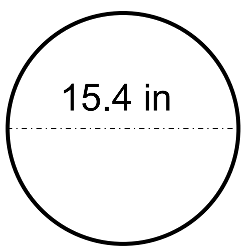
If r is the radius of a circle, then the area, A, of the circle is

$A = \pi r^2$

Find the circumference and the area of the following:

(For calculations involving π, use the π key from a scientific calculator).

Ex. 1



Solution:

We will use $C = \pi d$ to find the circumference since we are given the diameter:

$C = \pi d = \pi(15.4) = 15.4\pi$ in

= = 48.38… in ≈ 48.4 in.

To find the area, first divide 15.4 by 2 to get the radius.

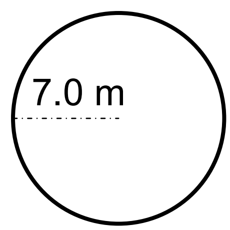
$r = 15.4 \div 2 = 7.7$ in.

Now use $A = \pi r^2$:

$A = \pi(7.7)^2 = 59.29\pi \text{ in}^2$

$= 186.2650\ldots \text{in}^2 \approx 186 \text{ in}^2$.

Ex. 2



Solution:

We will use $C = 2\pi r$ to find the circumference since we are given the radius:

$C = 2\pi r = 2\pi(7) = 14\pi$ m

= 43.982… m ≈ 44 m.

To find the area, replace r by 7:

$A = \pi r^2 = \pi(7)^2 = 49\pi \text{ m}^2$

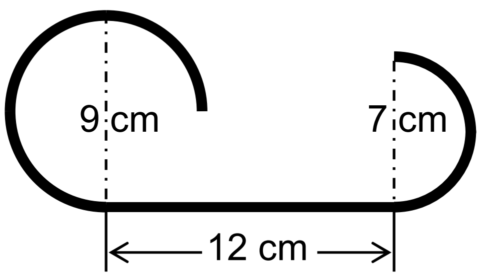
$= 153.938\ldots \text{ m}^2 \approx 150 \text{ m}^2$.

Objective 2: Working with Portions of Circles.

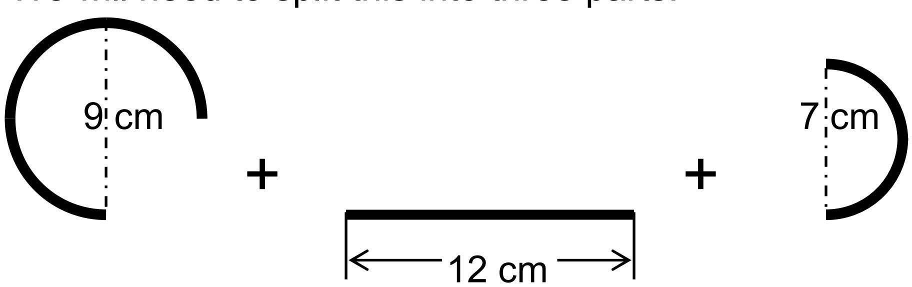
When bending a rod, the material on the inside part of the rod is compressed while the material on the outside part is stretched. In terms of calculating the radius or diameter of a circular portion, we must use the midline of the rod. For instance, if the inner radius of a ring is 9 mm and the outer radius is 11 mm, then the thickness of the rind is 2 mm. The midline radius is the average of the two radii, which is 10 mm. For our calculations involving the circumference, we will need to use 10 mm as our radius.

<u>**Find the length of stock needed to create the following shapes. Assume the stock is 0.5 cm thick. Round to one decimal place:**</u>

Ex. 3

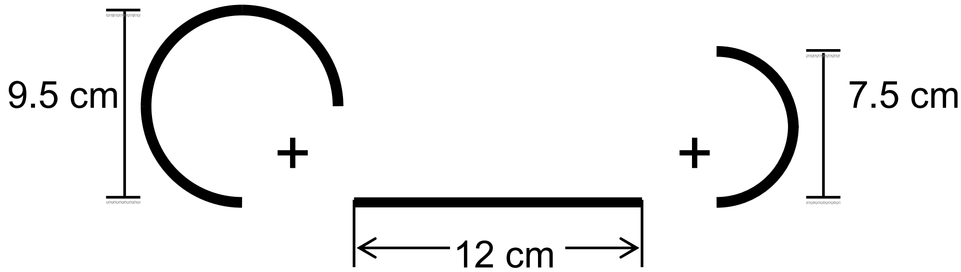


<u>Solution:</u>
We will need to split this into three parts:



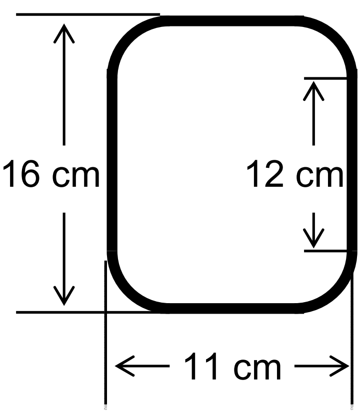
The first part is three-fourths of a circle with an inner diameter of 9 cm. Since the stock is 0.5 cm, then the outer diameter is 9 + 0.5 + 0.5 = 10 cm. Thus, the midline diameter is the average of 9 and 10 which is 9.5 cm. Hence, the length of the first part is three-fourths of the circumference of a circle with a diameter of 9.5 cm.
The length of the second part is 12 cm.
The third part is one-half of a circle with an inner diameter of

7 cm. Since the stock is 0.5 cm, then the outer diameter is
$7 + 0.5 + 0.5 = 8$ cm. Thus, the midline diameter is the average of 7 and 8 which is 7.5 cm. Hence, the length of the third part is one-half of the circumference of a circle with a diameter of 7.5 cm.

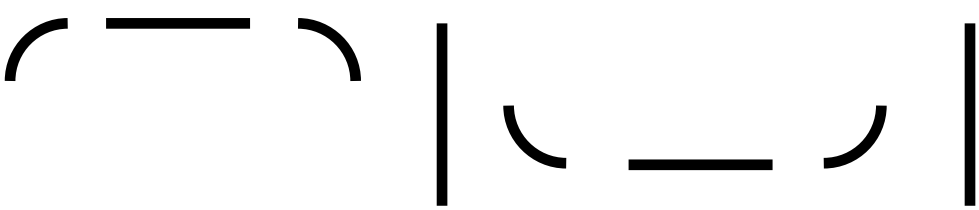


$$\frac{3}{4}\pi d \quad + \quad 12 \quad + \quad \frac{1}{2}\pi d$$
$$= \frac{3}{4}\pi(9.5) + 12 + \frac{1}{2}\pi(7.5) = 7.125\pi + 12 + 3.75\pi$$
$$= 10.875\pi + 12 = 34.1648\ldots + 12 = 46.1648\ldots \text{ cm}$$
$$\approx 46.2 \text{ cm}$$

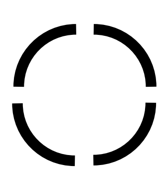
Ex. 4



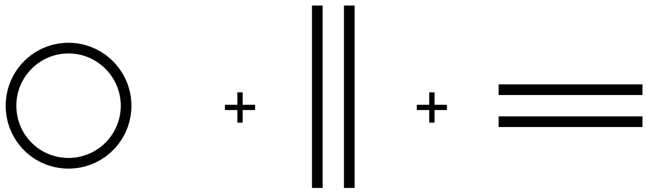
Solution:
We need to split this into eight parts:



Notice that the four portions of the circle form a complete circle: The outer diameter of the circle



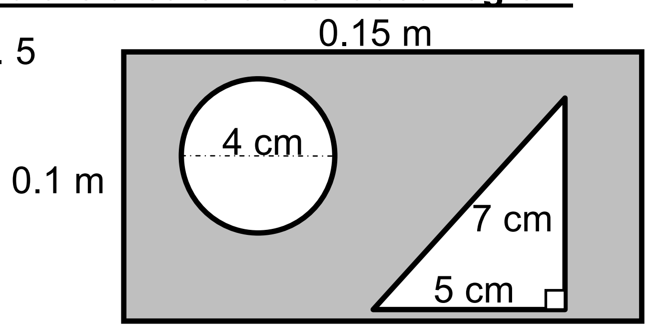
is 16 cm – 12 cm = 4 cm. Since the width of the stock is 0.5 cm. the inner diameter is 4 cm – 0.5 cm – 0.5 cm = 3 cm. Hence, the midline diameter is (4 + 3)/2 = 3.5 cm. The length of the base between the two quarter circles is equal to the length of the total base minus the outer diameter of the circle: 11 – 4 = 7cm
Thus, the amount of stock is equal to



$= \pi(3.5) + 2(12) + 2(7) = 3.5\pi + 24 + 14 = 10.9955\ldots + 24 + 14$
$= 48.9955\ldots \text{ cm} \approx 49.0 \text{ cm}.$

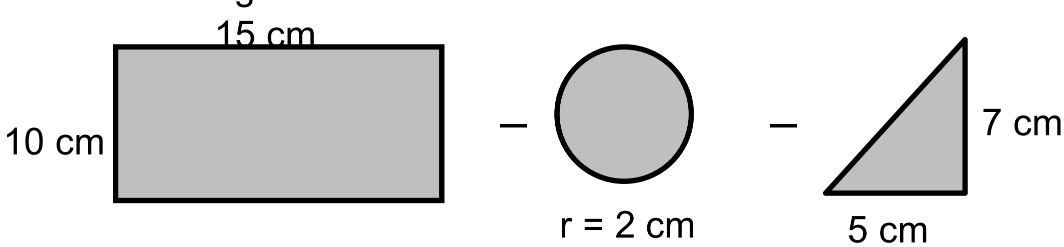
<u>Find the area of the shaded region:</u>

Ex. 5



<u>Solution:</u>
First convert the meters into centimeters:
0.1 m = 0.10 = 10 cm and 0.15 m = 0.15 = 15 cm

The area of the shade region is equal to the area of the rectangle minus the area of the circle (the radius is 4 ÷ 2 = 2 cm) and the area of the triangle:



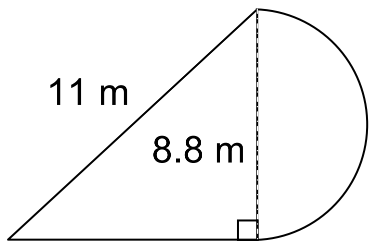
$A = Lw \qquad - \pi r^2 \qquad - \frac{1}{2}bh$

$= (15)(10) \qquad - \pi(2)^2 \qquad - \frac{1}{2}(5)(7)$

$= (15)(10) \ - \pi(2)^2 - \frac{1}{2}(5)(7) = 150 - 4\pi - 17.5 = 132.5 - 4\pi$
$= 119.93362\ldots\ cm^2 \approx 119.9\ cm^2$

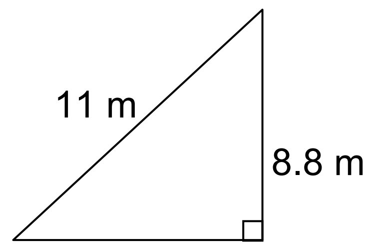
Find the perimeter and area of the following:

Ex. 5



Solution:

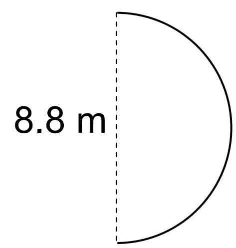
To find the perimeter, we will first need to find the base of the right triangle. Using the Pythagorean Theorem:



$$(11)^2 = (8.8)^2 + b^2$$
$$121 = 77.44 + b^2$$
$$\underline{-77.44 = -77.44}$$
$$43.56 = b^2$$

$b = \pm\sqrt{43.56} = \pm 6.6$ m.

So, the base is 6.6 m.

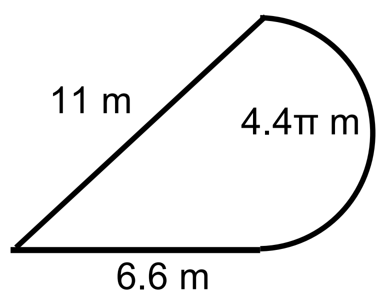


Next, we will need to find the length of the semicircle. For a full circle, the circumference is $C = \pi d$. Since we have a semicircle, we will divide our answer by 2:

$C = \pi d = \pi(8.8) = 8.8\pi$ m. Dividing by two, we get: $8.8\pi \div 2 = 4.4\pi$ m.

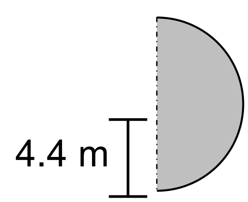
Now, we add up the sides :

$P = 6.6 + 11 + 4.4\pi = 17.6 + 4.4\pi$
$= 17.6 + 13.823\ldots \approx 31.423\ldots \approx 31.4$ m

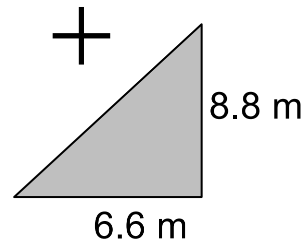


For the area, we will add the area of the triangle to the area of the semicircle. Since the diameter of the circle is 8.8 m, its radius is $8.8 \div 2 = 4.4$ m. The area for a circle is $A = \pi r^2$, so we will divide the answer by 2:

$\pi r^2 = \pi(4.4)^2 = 19.36\pi\ m^2$. Dividing by 2, we



get: $19.36\pi \div 2 = 9.68\pi$ m^2. Since the base is 6.6 m and the height is 8.8 m, the area of the triangle is $\frac{1}{2}(6.6)(8.8) = 29.04$ m^2.
Thus, the total area is
$9.68\pi + 29.04 = 30.4106\ldots + 29.04$
$= 59.4506\ldots \approx 59.5$ m^2.

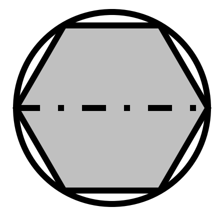


Solve the following:

Ex. 6 A machinist needs to mill a regular hexagon with each side 0.4 cm long from a circular piece of stock. What is the minimum diameter of the circular piece of stock to be used?

Solution:

We need to find the smallest circle that can enclose a regular hexagon with each side 0.4 cm long. The longest dimension in a regular hexagon is the distance across the corners (the diagonal).

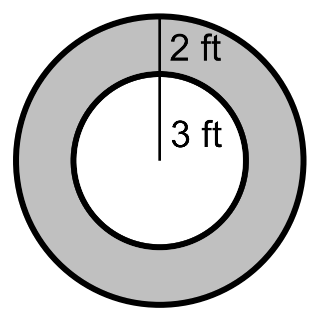


But that value is twice the distance of the length of each side:
d = 2a = 2(0.4) = 0.8 cm. The minimum diameter of the circular piece of stock has to be equal to the length of the diagonal. Hence, the minimum diameter is 0.8 cm.

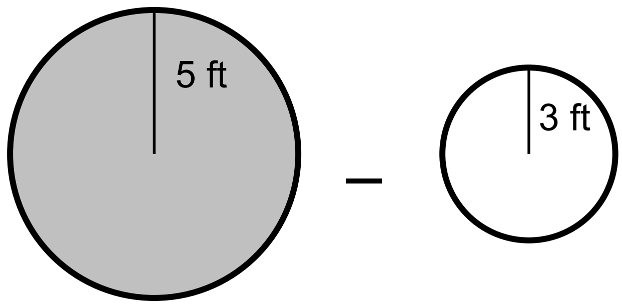
Ex. 7 Felix is install a water fountain with a circular base having a radius of 3.0 feet. A walkway with a width of 2.0 feet will surround the fountain. How many square yards of concrete will be needed for the walkway?

Solution:

We need to take the area of the fountain with the walkway and subtract the area of the fountain by itself to find the area of the concrete needed. The radius of the fountain with the walkway is equal to 3 ft + 2 ft = 5 ft.



Now, we can find the areas of the two circles and subtract to get our answer:



$$\pi r^2 \qquad - \qquad \pi r^2$$
$$= \pi(5)^2 \qquad - \qquad \pi(3)^2$$
$$= 25\pi - 9\pi = 16\pi = 50.265\ldots\ \text{ft}^2$$

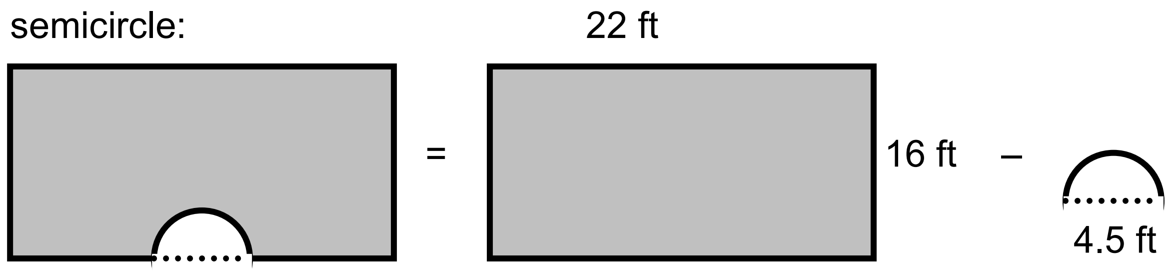
Now, we need to convert the answer to yd^2:

$$50.265\ldots\ \text{ft}^2 = \frac{50.265\ldots\ \text{ft}^2}{1} \bullet \frac{1\text{yd}^2}{9\text{ft}^2} = 5.585\ldots\ \text{yd}^2 \approx 5.6\ \text{yd}^2$$

Ex. 8 In building a 22-foot by 16-foot rectangular deck, Pat will need cut a semicircle of diameter of 4.5 feet out of one of the longer ends of the deck to accommodate a tree. If Pat charges \$12.50 per square foot plus \$100 fee for accommodating the tree, what is the total charge of the building the deck?

<u>Solution:</u>

We need to find the area of the deck and subtract the area of the semicircle:



The diameter of the semicircle is 4.5 ft, so its radius is 4.5 ÷ 2 = 2.25 feet. Hence, the area of the deck is:

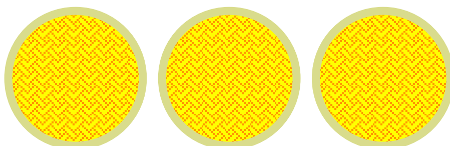
$$Lw \qquad - \qquad \frac{1}{2}\pi r^2$$
$$= 22(16) - \frac{1}{2}\pi(2.25)^2 = 352 - 2.53125\pi = 352 - 7.9521\ldots$$
$$= 344.047\ldots\ \text{ft}^2$$

The cost to build the deck is \$100 + \$12.50(number of square feet) = 100 + 12.50(344.047…) = 100 + 4300.598… = 4400.598… ≈ \$4,400.

Ex. 9 For the same price, Rosa can buy three twelve-inch pizzas or two sixteen-inch pizzas. a) Which set of pizzas is the better buy?
b) What percent (to the nearest tenth) more pizza does she get with the better buy?

<u>Solution:</u>

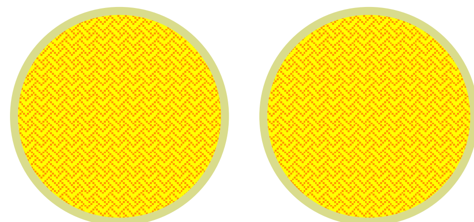
a) For pizza, we will need to find the total area of the three 12-in pizzas and compare it to the total area of the two 16-in pizzas. The set of pizzas with the most area will be the better buy. The size of the pizzas refers to its diameter. So, the radius of the 12-in pizza is 6 in and the radius of the 16-in pizza is 8 in. Now, we can calculate the area of each set:



The area of a circle is $A = \pi r^2$. For the medium set of pizzas, we will replace r by 6 in and calculate the area. Since we have three pizzas, we will multiply the answer by three to get the total area:

$$A = \pi r^2 = \pi(6)^2 = 36\pi \text{ in}^2.$$

Multiplying by 3, we get:
$3 \bullet 36\pi = 108\pi \text{ in}^2$.



For the large set of pizzas, we will replace r by 8 in and calculate the area. Since we have two pizzas, we will multiply the answer by two to get the total area:

$$A = \pi r^2 = \pi(8)^2 = 64\pi \text{ in}^2.$$

Multiplying by 2, we get:
$2 \bullet 64\pi = 128\pi \text{ in}^2$.

Since $128\pi \text{ in}^2 > 108\pi \text{ in}^2$, the two large pizzas are a better buy.

b) The two large pizzas offer $128\pi - 108\pi = 20\pi \text{ in}^2$ more area than the three medium pizzas. We want to find what percent of 108π is 20π:

$20\pi = P(108\pi)$

$20\pi = (108\pi)P$ (Divide both sides by 108π)

$\frac{20\cancel{\pi}}{108\cancel{\pi}} = \frac{(108\pi)P}{108\pi}$ (The π's divide out)

$P = 0.18518 \ldots = 18.518 \ldots \%$

$\approx 18.5\%$.