Sect 7.8 – Scientific Notation

Objective 1: Introduction to Scientific Notation

In chemistry, there are approximately 602,204,500,000,000,000,000,000 atoms per mole and in physics, an electron weighs approximately 0.000000000000000000000000000000911 kg. These numbers are not easy to represent in our present notation because they are either very large or very small. Yet, these types of numbers occur in the real world all the time, so a shorthand way called **Scientific Notation** was developed to express these numbers in a more compact fashion. The idea behind scientific notation is that any number can be written as a number between one and ten, including one, times a power of ten. Recall the powers of ten:

$10^0 = 1$ $\quad 10^1 = 10$ $\quad 10^2 = 100$

$10^3 = 1000$ $\quad 10^4 = 10000$ $\quad 10^5 = 100000$

Etc.

Also, since $x^{-n} = \frac{1}{x^n}$, then:

$10^{-1} = \frac{1}{10}$ $\quad 10^{-2} = \frac{1}{10^2} = \frac{1}{100}$ $\quad 10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$

$10^{-4} = \frac{1}{10^4} = \frac{1}{10000}$ $\quad 10^{-5} = \frac{1}{10^5} = \frac{1}{100000}$ Etc.

<u>Write each number as a number between one and ten times a power of ten:</u>

Ex. 1a 5000 Ex. 1b 43,900

Ex. 1c 0.0008 Ex. 1d 0.00000765

<u>Solution:</u>

a) $5{,}000 = 5 \times 1000 = 5 \times 10^3$

b) $43{,}900 = 4.39 \times 10000 = 4.39 \times 10^4$

c) $0.0008 = 8 \times .0001 = 8 \times \frac{1}{10000} = 8 \times \frac{1}{10^4} = 8 \times 10^{-4}$

d) $0.00000765 = 7.65 \times \frac{1}{1000000} = 7.65 \times \frac{1}{10^6} = 7.65 \times 10^{-6}$

Definition: A number of the form $W \times 10^n$ is in **Scientific Notation** if n is integer and $1 \leq |W| < 10$ (in other words, W has one non-zero digit to the left of the decimal point). The power 10^n is the **order of magnitude** of the number. The "×" means multiplication, not the variable x.

There are some shortcuts to figure out how to write the number in Scientific Notation. If the original number is ten or larger, the exponent n will be positive. If the original number is smaller than one, the exponent n will be negative. If the number is at least one, but smaller than 10, n will be zero. Also, the number of places you have to move the decimal point to find W will be equal to the power of ten.

A memory aid is # # # # # = $\underline{\#}$ **.** $\underline{\#\,\#\,\#\,\#}$ $\times 10^{\square}$.

For example, with 43,900, we know that n will be positive since 43,900 is bigger than 10. Since the decimal point is after the last zero, we have to move it four places until it is after the four. Thus 43,900 = 4.39×10^{4}. The order of magnitude of 4.39×10^{4} is 10^{4}. For 0.00000765, the decimal point is after the first zero so we have to move it six places until it is after the seven (remember W has only one non-zero digit to the left of the decimal point). Since 0.00000765 < 1, 0.00000765 = 7.65×10^{-6}.

Objective 2: Writing Numbers in Scientific Notation.

Write each number in scientific notation:

Ex. 2a	186,200	Ex. 2b	0.000342
Ex. 2c	0.00004563	Ex. 2d	– 45,879,000
Ex. 2e	602,204,500,000,000,000,000,000 atoms per mole		
Ex. 2f	0.000000000000000000000000000000911 kg		
Ex. 2g	– 5.43	Ex. 2h	0.32

Solution:

a) Since 186,200 > 10, the exponent will be positive. We have to move the decimal point five places to the left.
186,200 = 1.862×10^{5}

b) Since 0.000342 < 1, the exponent will be negative. We have to move the decimal point four places to the right.
0.000342 = 3.42×10^{-4}

c) Since 0.00004563 < 1, the exponent will be negative. We have to move the decimal point five places to the right.
0.00004563 = 4.563×10^{-5}

d) Since 45,879,000 > 10, the exponent will be positive. We have to move the decimal point seven places to the left.
$-45{,}879{,}000 = -4.5879 \times 10^7$

e) 602,204,500,000,000,000,000,000 > 10, the exponent will be positive. We have to move the decimal point twenty-three places to the left.
$602{,}204{,}500{,}000{,}000{,}000{,}000{,}000 = 6.022045 \times 10^{23}$ moles per atom.

f) Since 0.000000000000000000000000000000911 < 1, the exponent will be negative. We have to move the decimal point thirty-one places to the right.
$0.000000000000000000000000000000911 = 9.11 \times 10^{-31}$ kg.

g) Since 5.43 is between 1 and 10, the exponent will be 0.
$-5.43 = -5.43 \times 10^0$. We do not usually write single digit numbers in scientific notation.

h) Since 0.32 < 1, the exponent will be negative. We have to move the decimal point one place to the right.
$0.32 = 3.2 \times 10^{-1}$

Objective 3: Writing Numbers in Standard (Decimal) Notation

To convert a number in scientific notation to standard notation, we work backwards. If the power of 10 is positive, the number has to be greater than 10, so we move the decimal point that many places to the right. If the power of 10 is negative, the number has to be less than 1, so we move the decimal point that many places to the left.

Convert the following into standard notation:

Ex. 3a 4.5×10^3

Ex. 3b 6.7552×10^{-7}

Ex. 3c 4.093×10^{-3}

Ex. 3d -7.8×10^6

Solution:

a) Move the decimal point 3 places to the right.
$4.5 \times 10^3 = 4{,}500$

b) Move the decimal point 7 places to the left.
$6.7552 \times 10^{-7} = 0.00000067552$

c) Move the decimal point 3 places to the left.
$4.093 \times 10^{-3} = 0.004093$

d) Move the decimal point 6 places to the right.
$-7.8 \times 10^{6} = -7{,}800{,}000$

Objective 4: Entering Numbers in Scientific Notation on a Calculator.

On most scientific calculators, you will see either an EE key or an EXP key. This key let's us enter numbers in scientific notation. Let's try entering the following numbers in the calculator.

	Number	Keystrokes	Display
Ex. 4a	5.7×10^{11}	5.7 $\boxed{\text{EE}}$ 11 $\boxed{=}$	5.7 ^11^
Ex. 4b	-6.28×10^{15}	– 6.28 $\boxed{\text{EE}}$ 15 $\boxed{=}$	– 6.28 ^15^
Ex. 4c	8.5×10^{-12}	8.5 $\boxed{\text{EE}}$ – 12 $\boxed{=}$	8.5 ^– 12^
Ex. 4d	-4.7×10^{-5}	– 4.7 $\boxed{\text{EE}}$ – 5 $\boxed{=}$	– 0.000047

Notice that with the last example, the calculator converted the number to standard notation. Most scientific calculators will convert the number into standard notation if it is possible to display the number. If we want our number to remain in scientific notation, we will need to change the mode from FLO to SCI. On a TI-30XA, FLO is the yellow above the 4 and SCI is the yellow above the 5. To change the mode to SCI on a TI-30XA, hit the 2nd key and then 5:

Ex. 4d – 0.000047 $\boxed{\text{2nd}}$ $\boxed{5}$ $\boxed{=}$ – 4.7 $^{-05}$

This gives you a nice way to convert from standard notation to scientific notation. However, now you calculator will display all the answers in scientific notation. Try multiplying 23 and 726 on your calculator:

Ex. 5 23 $\boxed{\text{x}}$ 726 $\boxed{=}$ 1.6698 ^{04}

To get our calculator back to "normal" so to speak, we will need to change the mode back to floating point (FLO). To change the mode to FLO on a TI-30XA, hit the 2nd key and then 4:

Ex. 5 1.6698 ^{04} $\boxed{\text{2nd}}$ $\boxed{4}$ $\boxed{=}$ 16698

Now you calculator will work the way you expected it to work. This also gives us a nice way to convert from scientific notation to standard notation provided that the calculator can display the result in standard notation.

Objective 5: Multiplying and Dividing Numbers in Scientific Notation.

Multiplication and division of numbers in scientific notation is relatively straight forward. We use the commutative and associative properties of multiplication to group the numbers together and group the powers of 10 together and then simplify. The thing one has to watch for is after computing the result, the number in front of the power of ten may not be between 1 and ten (including one). In this case, some additional converting and simplifying will need to be performed to get the answer in scientific notation.

Simplify the following, Be sure your answer is in scientific notation. Then perform the operation using a scientific calculator:

Ex. 6a $(4.3 \times 10^{15})(1.8 \times 10^{23})$ Ex. 6b $\frac{9.6\times10^{-57}}{2.4\times10^{-13}}$

Ex. 6c $\frac{1.89\times10^{24}}{9\times10^{7}}$ Ex. 6d $(6.022 \times 10^{23})(9.11 \times 10^{-31})$

Solution:

a) $(4.3 \times 10^{15})(1.8 \times 10^{23})$ (regroup and reorder)

$= 4.3\bullet1.8 \times 10^{15}\bullet10^{23}$ (product rule & simplify)

$= 7.74 \times 10^{38} \approx 7.7 \times 10^{38}$

b) $\frac{9.6\times10^{-57}}{2.4\times10^{-13}}$ (regroup and reorder)

$= \frac{9.6}{2.4} \times \frac{10^{-57}}{10^{-13}}$ (quotient rule)

$= \frac{9.6}{2.4} \times 10^{-57-(-13)}$ (simplify)

$= 4.0 \times 10^{-44}$

c) $\frac{1.89 \times 10^{24}}{9 \times 10^{7}}$ (regroup and reorder)

$= \frac{1.89}{9} \times \frac{10^{24}}{10^{7}}$ (quotient rule)

$= \frac{1.89}{9} \times 10^{24-7}$ (simplify)

$= 0.21 \times 10^{17}$, but this is not in scientific notation.
However, $\mathbf{0.21 = 2.1 \times 10^{-1}}$,
So, 0.21×10^{17}
$= \mathbf{2.1 \times 10^{-1}} \times 10^{17}$ (product rule)
$= 2.1 \times 10^{16} \approx 2 \times 10^{16}$

d) $(6.022 \times 10^{23})(9.11 \times 10^{-31})$ (regroup and reorder)
$= 6.022 \bullet 9.11 \times 10^{23} \bullet 10^{-31}$ (product rule & simplify)
$= 54.86042 \times 10^{-8}$, but this is not in scientific notation.
However, $\mathbf{54.86042 = 5.486042 \times 10^{1}}$.
So, 54.86042×10^{-8}
$= \mathbf{5.486042 \times 10^{1}} \times 10^{-8}$ (product rule)
$= 5.486042 \times 10^{-7} \approx 5.49 \times 10^{-7}$

Objective 6: Applications

Ex. 7 If the mass of a proton is approximately 1.67×10^{-24} grams, how much mass would 6.022×10^{23} protons have?

<u>Solution:</u>

We need to multiply the number of protons by the mass of each proton:

$(6.022 \times 10^{23})(1.67 \times 10^{-24})$ (regroup and reorder)
$= 6.022 \bullet 1.67 \times 10^{23} \bullet 10^{-24}$ (product rule & simplify)
$= 10.05674 \times 10^{-1}$, but this is not in scientific notation.
However, $\mathbf{10.05674 = 1.005674 \times 10^{1}}$.
So, 10.05674×10^{-1}
$= \mathbf{1.005674 \times 10^{1}} \times 10^{-1}$ (product rule)
$= 1.005674 \times 10^{0} \approx 1.01$ grams

Ex. 8 A star called "Vega" is approximately 1.552×10^{14} miles from the Earth. If the space ship can travel 2.100×10^{4} miles per hour, how many years will it take the space ship to reach Vega?

Solution:

First, we will calculate how far the space ship can travel in a year:

Since 24 hours = 2.4×10^1 and 365 = 3.65×10^2, then

2.1×10^4 miles/hour (multiply by 2.4×10^1 hours in a day)

$= (2.1 \times 10^4)(2.4 \times 10^1)$ (regroup and reorder)

$= 2.1 \bullet 2.4 \times 10^4 \bullet 10^1$ (simplify)

$= 5.04 \times 10^5$ miles/day (multiply by 3.65×10^2 days in a year)

$= (5.04 \times 10^5)(3.65 \times 10^2)$ (regroup and reorder)

$= 5.04 \bullet 3.65 \times 10^5 \bullet 10^2$ (simplify)

$= 18.396 \times 10^7$

But, $\mathbf{18.396 = 1.8396 \times 10^1}$

So, 18.396×10^7

$= \mathbf{1.8396 \times 10^1} \times 10^7$

$= 1.8396 \times 10^8$ miles/year

Since d = rt (distance equals rate times time), we replace d by 1.552×10^{14} miles and r by 1.8396×10^8 miles/year and solve:

d = rt

$$\frac{1.552 \times 10^{14}}{1.8396 \times 10^8} = \frac{(1.8396 \times 10^8)t}{1.8396 \times 10^8}$$ (divide both sides by 1.8396×10^8. regroup and reorder)

$$t = \frac{1.552}{1.8396} \times \frac{10^{14}}{10^8}$$ (simplify)

$\approx 0.8436 \ldots \times 10^6$

But, $\mathbf{0.8436\ldots = 8.436.. \times 10^{-1}}$

So, $0.8436\ldots \times 10^6$

$= \mathbf{8.436\ldots \times 10^{-1}} \times 10^6$

$= 8.436\ldots \times 10^5$ years $\approx 8.4 \times 10^5$ years

It will take the space ship approximately 8.4×10^5 or 840,000 years to reach the star "Vega."