Sect 7.6 – Solving Word Problems

Objective 1: Understanding Key Words and their Translations.

Here is a list of key words and their translation:

Addition

the total of 11 and x	(11 + x)
8 added to y	y + 8
7 more than f	f + 7
4 increased by w	4 + w
the sum of 8 and h	(8 + h)
r plus s	r + s
13 greater than t	t + 13
exceeds r by 6	r + 6

Subtraction

6 minus x	6 – x
4 less y	4 – y
4 less than y	y – 4
the difference between 8 and x	(8 – x)
y decreased by 5	y – 5
x subtract 5	x – 5
x subtracted from 5	5 – x
q reduced by 43	q – 43

Multiplication

6 times d	6d
the product of 11 and r	(11r)
$\frac{2}{5}$ of J	$\frac{2}{5}$ J
V multiplied by $\frac{5}{6}$	$\frac{5}{6}$ V
twice R	2R
triple w	3w
double q	2q

Division

7 divided into x	$x \div 7 = \frac{x}{7}$
7 divided by x	$7 \div x = \frac{7}{x}$
the quotient of 5 and y	$(5 \div y) = \frac{5}{y}$
the ratio of 4 to 11	$4 \div 11 = \frac{4}{11}$
y split into 7 equal parts	$y \div 7 = \frac{y}{7}$
x over 4	$\frac{x}{4}$
9 per y	$9 \div y = \frac{9}{y}$

Note: Subtraction and division are not commutative. You need to learn the correct order with the vocabulary word.

Objective 2: Writing an equation from an application.

Recall that the words: is, is equals to, yields, amounts to, the same as, gives, makes, leaves, the result is and equals all mean "=." Let us try some examples of translating phrases into equations.

Write each word phrase in symbols:

Ex. 1 The distance a car travels is equal to the product of the speed and the time.

Solution:

Let d = the distance
r = the speed
t = the time

The word product means multiply, so

$d = r \bullet t$ or $d = rt$

Ex. 2 The sum of the principal and interest is the total amount in the account.

Solution:

Let p = the principal
i = the interest
a = the total amount

The word total means add, thus

$a = p + i$

Ex. 3 The water pressure (psi) is the result of 62.4 multiplied by the depth of the water (ft).

Solution:

Let P = the water pressure

d = the depth of the water

So, $P = 62.4d$

Ex. 4 The amount of chicken needed for a banquet is one third more than double the amount of beef needed.

Solution:

Let c = the amount of chicken

b = the amount of beef

The word one third more than means the amount of beef times 2 plus 1/3 of the amount of beef: $c = 2b + \frac{1}{3}b$ or $c = \frac{7}{3}b$

Ex. 5 The quotient of pi times the diameter of a cylindrical object times the speed of the rotation of the object and twelve will yield the surface speed of the rotating object.

Solution:

Let d = the diameter of the cylindrical object

r = the speed of the rotation

s = the surface speed

We need to break this down into pieces.

"pi times the diameter of a cylindrical object times the speed of the rotation of the object:" πdr

"The quotient of" πdr "and twelve will yield the surface speed…"

$$s = \frac{\pi dr}{12}$$

Ex. 6 The wall thickness of a pipe is the same as half of the difference of the outer diameter of the pipe and the inner diameter of the pipe.

Solution:

Let T = the wall thickness of the pipe

D = the outer diameter

d = the inner diameter

"difference of the outer diameter … and the inner diameter …:" (D – d)

"The wall thickness of a pipe is the same as half of" (D – d):

$$T = \frac{1}{2}(D - d)$$

Ex. 7 The diameter of the crankshaft gear is equal to the sum of twice the height of the teeth above the pitch diameter of the small gear and the pitch diameter of the small gear.

Solution:

Let c = the diameter of the crankshaft gear
h = the height of the teeth above the pitch diameter
p = the pitch diameter
" twice the height of the teeth…:" 2h
" the sum of" 2h "and the pitch diameter …:" 2h + p
Thus, c = 2h + p

Ex. 8 The net resistance of two resistors in parallel is the same as the product of the resistance of the two resistors divided by the total of the resistance of the two resistors.

Solution:

Let R_P = the net resistance of two resistors in parallel
R_1 = the resistance of the first resistor.
R_2 = the resistance of the second resistor
"the product of the resistance of the two resistors:" R_1R_2
"total of the resistance of the two resistors:" $(R_1 + R_2)$
"The net resistance … is the same as" R_1R_2 "divided by" $(R_1 + R_2)$:

$$R_P = \frac{R_1R_2}{R_1+R_2}$$

Objective 2: Solving Application Problems

When solving application problems, it is important to be organized and systematic in our approach. Here are some guidelines to help us to do so:

Problem Solving

1) Read the problem - Overview - Identify the type of problem
2) Read the problem - Phrase by Phrase - Ask yourself what each phrase means.
3) Read the problem - Begin to write things down. Label known and unknown values. Write all unknowns using one variable
4) Draw a chart, a diagram, or a picture. Label it appropriately. You may need to get a formula.
5) Translate the problem into an equation and solve.
6) Check.
   i) Reality Check - does it make sense?
   ii) Did you answer the questions posed by the problem?

Set-up the equation and solve the following:

Ex. 9 Juan buys three CDs for every DVD he purchases. If the cost of a CD is \$15.99 and the cost of DVD is \$19.99, how many CDs and DVDs does he purchase if he spends \$271.84?

Solution:

Let D = the number of DVDs he buys.

Now, write the number of CDs is terms of the number of DVDs:

"Juan buys three CDs for every DVD he purchases"

Thus, if Juan buys 1 DVD, he will buy 3 CDs. If he buys 2 DVDs, he will buy 6 CDs. If he buys D number of DVDs, he will buy 3•D number of CDs.

3D = the number of CDs he buys.

Since the total spent for each type of item is the price times the number purchased, then the amount he spends on DVDs is 19.99•D and the amount he spends on CDs is 15.99•3D = 47.97D. The total amount spent is:

amount spent on DVDs + amount spent on CDs = 271.84

19.99D + 47.97D = 271.84

Thus, our equation is:

$19.99D + 47.97D = 271.84$ (combine like terms)

$\frac{67.96D}{67.96} = \frac{271.84}{67.96}$ (divide by 67.96)

$D = 4$

Also, $3D = 3(4) = 12$.

Juan bought 4 DVDs and 12 CDs.

Ex. 10 A mechanic is paid \$26 per hour. If she works overtime (more than 40 hours), she is paid time and a half for each hour she works overtime. In one week, she earned \$1703. How many hours did she work that week?

Solution:

Let h = the number of hours she worked during the week.

The number of hours overtime is the number of hours subtract 40:

h – 40 = the number of hours of overtime

She gets paid time and a half for each hour of overtime. Thus, she gets \$26 plus an additional $\frac{1}{2}$•26 = \$13 per hour for every hour of overtime.

So, the equations is:

Pay at the regular rate	+	Additional pay per hour of overtime	=	Total Pay
26h	+	13(h – 40)	=	1703

Hence, our equation is:

$26h + 13(h - 40) = 1703$ (distribute)
$26h + 13h - 520 = 1703$ (combine like terms)
$39h - 520 = 1703$ (add 520 to both sides)
$+ 520 = + 520$
$\frac{39h}{39} = \frac{2223}{39}$ (divide by 39)
$h = 57$

She worked a total of 57 hours.

Ex. 11 A rectangular foundation of a building has a perimeter of 144 feet. If the length is twenty-four feet less than twice the width, find the dimensions of the garden.

Solution:

Since the length is described in terms of the width, we want to let the variable w represent the width and write the length in terms of w:

Let w = the width of the rectangle

Since "length is twenty-four feet less than twice the width," then

$2w - 24$ = the length of the rectangle

The formula for the perimeter of a rectangle is:

$P = 2L + 2w$

Replace P by 144 feet and L by (2w – 24):

$144 = 2(2w - 24) + 2w$ (distribute)
$144 = \mathbf{2}(2w) - \mathbf{2}(24) + 2w$ (multiply)
$144 = 4w - 48 + 2w$ (combine like terms)
$144 = 6w - 48$

Now, solve for w:

$144 = 6w - 48$ (add 48 to both sides)
$+ 48 = + 48$
$\frac{192}{6} = \frac{6w}{6}$ (divide by 6)
$32 = w$

Plug 32 into the expression 2w – 24 to find the length:

$2(32) - 24 = 64 - 24 = 40$.

So, the length of the foundation is 40 feet and the width is 32 feet.

Ex. 12 Including 8.25% sales tax, the total cost to replace an air conditioning unit was $2381.50. If the materials cost was $1552 and the labor change was $54 per hour per person, how many hours did it take for two technicians to install the ac unit?

Solution:

Let h = the number of hours the two technicians worked.

Thus, the total labor hours on the job was 2h. Since the cost per hour was $54, then the total labor cost was 54•2h = 108h.

Labor Cost	+	Material Cost	=	Total before tax
108h	+	1552	=	Subtotal

Since sales tax is 8.25%, then the total paid is

Subtotal + 8.25%•Subtotal = 1•Subtotal + 0.0825•Subtotal

= 1.0825•Subtotal

Hence, 1.0825•Subtotal = 2381.50.

But. **Subtotal = 108h + 1552**, so our equation becomes:

1.0825(**108h + 1552**) = 2381.50 (distribute)

116.91h + 1680.04 = 2381.50 (subtract 1680.04)

– 1680.04 = – 1680.04

$\frac{116.91h}{116.91} = \frac{701.46}{116.91}$ (divide by 116.91)

h = 6

It took the technicians six hours to complete the job.

Ex. 13 To make concrete, one needs to mix 1 part cement, 2 parts water, 2 parts sand, and 3 parts gravel. If a parking lot will require 1900 ft^3 of concrete, how much of each ingredient will be used? Assume 5% will be lost to waste.

Solution:

Let c = the amount of cement

Since 2 parts of water are needed, then

2c = the amount of the water

Since 2 parts of sand are needed, then

2c = the amount of the sand

Since 3 parts of gravel are needed, then

3c = the amount of the gravel

The total mixture is

Cement	+	Water	+	Sand	+	Gravel	=	Concrete
c	+	2c	+	2c	+	3c	=	Concrete

But 5% will be lost due to waste:

Concrete – 5%•Concrete = Parking Lot

1•Concrete – 0.05•Concrete = Parking Lot

0.95•Concrete = Parking Lot

Hence, 0.95•Concrete = 1900

But **Concrete = c + 2c + 2c + 3c = 8c**

Thus,

$0.95 \bullet 8c = 1900$ (multiply)

$\frac{7.6c}{7.6} = \frac{1900}{7.6}$ (divide by 7.6)

$c = 250$

$2c = 500$

$3c = 750$

So, 250 ft^3 of cement, 500 ft^3 of water, 500 ft^3 of sand, and 750 ft^3 of gravel will be used for the parking lot.

Ex. 14 For a large convention, Katz's Katering Service determines that the number of pumpkin pies they will need is one sixth more than triple the number of mince meat pies they will need. If they need 114 pumpkin pies, how many mince meat pies will they need?

Solution:

Let p = the number of pumpkin pies

m = the number of mince meat pies

The word one sixth more than means the number of mince meat pies times 3 plus 1/6 of the number of mince meat pies:

$p = 3m + \frac{1}{6}m$ or $p = \frac{19}{6}m$

But, the number of pumpkins pies is 114, so our equation becomes:

$114 = \frac{19}{6}m$ (divide by $\frac{19}{6}$)

$\frac{114}{\frac{19}{6}} = \frac{\frac{19}{6}m}{\frac{19}{6}}$

$36 = m$

They will need 36 mince meat pies.