Sect 7.4 – Solving Two-Step Equations

Objective 1: Solving Linear Equations Involving Multiple Steps

In solving an equation involving more than one step, our goal is to isolate the variable terms on one side and the constant terms on the other side. We will do this by using the addition and subtraction properties first. Once we get the variable and constant terms isolated on opposite sides, we then use the multiplication and division properties to solve. Let's try some examples:

Solve and check the following:

Ex. 1 $-3x + 8 = -15$

Solution:

First subtract 8 from both sides, and then divide both sides by – 3.

$-3x + 8 = -15$

$\underline{\quad -8 = -8 \quad}$

$-3x = -23$

$\frac{-3x}{-3} = \frac{-23}{-3}$

$x = \frac{23}{3} = 7\frac{2}{3}$

Check:

$-3x + 8 = -15$

$-3(\frac{23}{3}) + 8 = -15$

$-23 + 8 = -15$

$-15 = -15$

True

Ex. 2 $-8.2 + 1.3x = -10.8$

Solution:

$-8.2 + 1.3x = -10.8$ (add 8.2 to both sides)

$\underline{+8.2 \quad = +8.2}$

$1.3x = -2.6$

$\frac{1.3x}{1.3} = \frac{-2.6}{1.3}$ (divide both sides by 1.3)

$x = -2$

Check:

$-8.2 + 1.3x = -10.8$

$-8.2 + 1.3(-2) = -10.8$

$-8.2 - 2.6 = -10.8$

$-10.8 = -10.8$ True

Ex. 3 $-\frac{4}{7}x + 36 = 92$

Solution:

$-\frac{4}{7}x + 36 = 92$ (subtract 36 from both sides)

$-36 = -36$

$-\frac{4}{7}x = 56$ (multiply both sides by the reciprocal of $-\frac{4}{7}$)

$-\frac{7}{4}\left(-\frac{4}{7}x\right) = -\frac{7}{4}\left(\frac{56}{1}\right)$

$x = -98$

Check:

$-\frac{4}{7}x + 36 = 92$

$-\frac{4}{7}\left(\frac{-98}{1}\right) + 36 = 92$

$56 + 36 = 92$

$92 = 92$ True

Ex. 4 $7x - 3 = 5x$

Solution:

To get the variable terms isolated on the opposite side from the constant term, we need to subtract 7x from both sides:

$7x - 3 = 5x$

$-7x = -7x$

$-3 = -2x$

Now, divide by -2

$\frac{-3}{-2} = \frac{-2x}{-2}$

$1.5 = x$

Check:

$7x - 3 = 5x$

$7(1.5) - 3 = 5(1.5)$

$10.5 - 3 = 7.5$

$7.5 = 7.5$

True

Objective 2: Simplifying Each Side of an Equation by Combining Like Terms.

When working with equations, we may need to simplify each side of the equation before solving. We want to look at each side in isolation and see if there any like terms that can be combined. Then we will proceed to solve it.

Solve the following:

Ex. 5 $3.2x - 5 + 4.1x - 8 = 2x - 7 - 2x$

Solution:

First, let us simplify the left side:

$3.2x - 5 + 4.1x - 8$ (combine like terms)

$= 7.3x - 13$

Now, let us simplify the right side:

$2x - 7 - 2x$ (combine like terms)

$= -7$

Thus, our equation becomes:

$7.3x - 13 = -7$ (add 13 to both sides)

$+13 = +13$

$7.3x = 6$

$\frac{7.3x}{7.3} = \frac{6}{7.3}$ (divide by 7.3)

$x = \frac{60}{73}$

Ex. 6 $9x + 15 - 9x = 5x - 7 + 8.2x - 22.1 + 12x$

Solution:

First, let us simplify the left side:

$9x + 15 - 9x$ (combine like terms)

$= 15$

Now, let us simplify the right side:

$5x - 7 + 8.2x - 22.1 + 12x$ (combine like terms)

$= 25.2x - 29.1$

Thus, our equation becomes:

$15 = 25.2x - 29.1$ (add 29.1 to both sides)

$+29.1 = +29.1$

$44.1 = 25.2x$

$\frac{25.2x}{25.2} = \frac{44.1}{25.2}$ (divide by 25.2)

$x = 1.75$

Ex. 7 $-\frac{8}{9}x - \frac{10}{9} + \frac{2}{3}x - \frac{7}{8}x = -\frac{17}{6} + \frac{5}{12}$

Solution:

First, let us simplify the left side:

$-\frac{8}{9}x - \frac{10}{9} + \frac{2}{3}x - \frac{7}{8}x$ (combine like terms)

$= -\frac{79}{72}x - \frac{10}{9}$

Now, let us simplify the right side:

$$-\frac{17}{6}+\frac{5}{12} \qquad \text{(combine like terms)}$$

$$=-\frac{29}{12}$$

Thus, our equation becomes:

$$-\frac{79}{72}x-\frac{10}{9}=-\frac{29}{12} \qquad \text{(add } \frac{10}{9} \text{ to both sides)}$$

$$+\frac{10}{9}=+\frac{10}{9}$$

$$-\frac{79}{72}x=-\frac{47}{36} \qquad \text{(divide by } -\frac{79}{72}\text{)}$$

$$x=\frac{94}{79}$$

Solve the following:

Ex. 8 If the surface area (SA) of a cylinder with a radius (r) of 5.6 meters is 520.7504 m^2, find the height (h). Use $\pi \approx 3.14$.
(hint: $SA = 2\pi r^2 + 2\pi rh$)

Solution:

$SA = 2\pi r^2 + 2\pi rh$ (replace r by 5.6, π by 3.14, and SA by 520.4864)

$520.7504 = 2(3.14)(5.6)^2 + 2(3.14)(5.6)h$ (exponents)

$520.7504 = 2(3.14)(31.36) + 2(3.14)(5.6)h$ (multiply)

$520.7504 = 196.9408 + 35.168h$

Now, let us solve the equation:

$520.7504 = 196.9408 + 35.168h$ (subtract 196.9408)

$-196.9408 = -196.9408$

$323.8096 = 35.168h$

$\frac{323.8096}{35.168} = \frac{35.168h}{35.168}$ (divide by 35.168)

$h = 9.2075\ldots$ (round to two significant digits)

$h \approx 9.2$ m

The height is 9.2 m.

Ex. 9 Maria invested $\$4\overline{00}0$ in an IRA. If, after two years her account balance was $4610.21, find the interest rate. (hint: A = p + prt)

Solution:

$A = p + prt$ (replace A by 4610.21, p by 4000, and t by 2)

$4610.21 = (4000) + (4000)r(2)$ (multiply)

$4610.21 = 4000 + 8000r$

Now, let's solve the equation:

$4610.21 = 4000 + 8000r$ (subtract 4000)

$-4000 = -4000$

$610.21 = 8000r$

$\frac{610.21}{8000} = \frac{8000r}{8000}$ (divide by 8000)

$r = 0.07627625$ (write as a percent)

$r = 7.627625\%$ (round to three significant digits)

$r \approx 7.63\%$

The interest rate is 7.63%.

Ex. 11 In certain situations, the length L of belt that connects two pulleys with diameters d and D that are distance s apart can be approximated by the formula: $L = 2s + 1.63(d + D)$

If two pulleys have diameters of 12 in and 28 in respectively, how far apart can the pulleys be if the length of the belt 168 in?

Solution:

$L = 2s + 1.63(d + D)$ (replace L by 168, d by 12 and D by 28)

$168 = 2s + 1.63(12 + 28)$ (add inside the parenthesis)

$168 = 2s + 1.63(40)$ (multiply)

$168 = 2s + 65.2$

Now, let's solve the equation:

$168 = 2s + 65.2$ (subtract 65.2)

$-65.2 = -65.2$

$102.8 = 2s$

$\frac{102.8}{2} = \frac{2s}{2}$ (divide by 2)

$s = 51.4$

$s \approx 51$ in

The two pulleys are 51 inches apart.