Sect 5.1 – Working with Measurement Numbers

Objective 1: Understanding Units

When making a measurement, whether an electrician is measuring the resistance in a circuit, a chef is measuring the amount of olive oil used in a recipe, or a plumber is measuring the length a pipe, it is important to include the units with the measurement.
A measurement has two components, the size and the units. For example, if a pipe is 12 ft., the size is 12 and the units are ft. In other words, 12 ft. is the same as 12×1 ft.

Objective 2: Using Significant Digits

All devices used in making measurements have a certain level of accuracy. We cannot use a measuring tape to measure a length of a table to be 110.2345262 inches. A measuring tape does not have that degree of accuracy. We could us the measuring tape to measure a table to be $110\frac{1}{4}$ inches. That measurement would fit within the degree of accuracy that a measuring tape can make.

To indicate the degree of accuracy of the measuring device being employed, we write the measurement using **significant digits**. There are three important rules that we follow to determine the number of significant digits in a measurement:

1) Any nonzero digit is significant.
2) The digit zero is significant if one of the following is true:
   a) It appears between two significant digits (i.e., 5**0**6 mm).
   b) It is at the right end of a decimal number (i.e., 2.7**0**)
   or c) It is marked with an overbar (i.e., $45\bar{\mathbf{0}}$)
3) The digit zero is **not** significant if one of the following is true:
   a) It is at the right end of a whole number (i.e., 340).
   or b) It is at the left end of a number (i.e., 0.002)

Determine the number of significant digits in each problem:

Ex. 1a	0.07 ohms	Ex. 1b	43.450 mm
Ex. 1c	8.09 amp	Ex. 1d	7,800 gal
Ex. 1e	300.0 in	Ex. 1f	$45\bar{0}0$ rpm

<u>Solution:</u>

a) 0.07 has one significant digit. The zero is not significant (3b).
b) 43.450 has five significant digits. The zero is significant (2b).
c) 8.09 has three significant digits. The zero is significant (2a).
d) 7,800 has two significant digits. The zeros are not significant (3a).
e) 300.0 has four significant digits The zeros are significant (2a & 2b).
f) $45\bar{0}0$ has three significant digits. The zero with the overbar is significant (2c) but the other zero is not (3a).

Objective 3: Understanding Accuracy and Precision

The accuracy of a measurement refers to the number of significant digits the measurement contains. In example 1b, 43.450 has five significant digits. We would say that 43.450 is accurate to five significant digits. Likewise in example 1e, 300.0 is accurate to four significant digits.

The precision of a measurement refers to how precise the measurement was made. This will be equal to the place value of the right-most significant digit. In example 1b, 43.450 is precise to the nearest thousandth whereas 300.0 is precise to the nearest tenth.

Accuracy – equals the number of significant digits of measurement has.
Precision – equals the place value of the right-most significant digit.

<u>Find the accuracy and precision of each number:</u>

Ex. 2a	5.08 oz	Ex. 2b	30 qt
Ex. 2c	2854 ft	Ex. 2d	0.0574 volts
Ex. 2e	78.1 lb	Ex. 2f	0.045 sec

<u>Solution:</u>

a) 5.08 is accurate to three significant digits. It is precise to the nearest hundredth.
b) 30 is accurate to one significant digit. It is precise to the nearest ten.
c) 2854 is accurate to four significant digits. It is precise to the nearest whole number.
d) 0.0574 is accurate to three significant digits. It is precise to the nearest ten - thousandth.

e) 78.1 is accurate to three significant digits. It is precise to the nearest tenth.

f) 0.045 is accurate to two significant digits. It is precise to the nearest thousandth.

Notice that 2854 ft is the most accurate number, but 0.0574 volts is the most precise.

Objective 4: Adding and Subtracting measurements

When adding and subtracting measurements with the same units, we always round the answer to the least precise measurement in the sum or difference. The units in our answer will be the same.

Simplify the following:

Ex. 3a	67.4 ft + 9.73 ft	Ex. 3b	0.95 sq in – 0.046 sq in
Ex. 3c	26 qt – 2.32 qt	Ex. 3d	93.7 sec + 22.812 sec
Ex. 3e	0.003 volts + $\frac{1}{4}$ volts	Ex. 3f	2.46 m – 0.006 m + 1.1 m

Solution:

a) Round the answer to the nearest tenth:
67.4 ft + 9.73 ft = 77.13 ft ≈ 77.1 ft.

b) Round the answer to the nearest hundredth:
0.95 sq in – 0.046 sq in = 0.904 sq in ≈ 0.90 sq in.

c) Round the answer to the nearest whole number:
26 qt – 2.32 qt = 23.68 qt ≈ 24 qt.

d) Round the answer to the nearest tenth:
93.7 sec + 22.812 sec = 116.512 sec ≈ 116.5 sec.

e) Round the answer to the nearest hundredth:
0.003 volts + $\frac{1}{4}$ volts = 0.003 volts + 0.25 volts
= 0.253 volts ≈ 0.25 volts

f) Round the answer to the nearest tenth:
2.46 m – 0.006 m + 1.1 m = 3.554 m ≈ 3.6 m

Objective 5: Multiplying and Dividing Measurements

Unlike addition and subtraction, when multiplying and dividing measurements, the units in the answer are not the same. Not only do we have to multiply or divide the numbers in the measurements, but we also have to multiply or divide the units.

Simplify the following:

Ex. 4a	4 in × 5 in	Ex. 4b	3 ft × 4 ft × 5 ft
Ex. 4c	80 mph × 5 hr	Ex. 4d	54 lb ÷ 1.8 in^2
Ex. 4e	56 sq ft ÷ 8 ft	Ex. 4f	105 m ÷ 35 m/sec

Solution:

a) 4 in × 5 in = (4 × 5) × (in × in) = 20 sq in. This is the area of a rectangle with a length of 4 in and a width of 5 in. The units for area are squared units.

b) 3 ft × 4 ft × 5 ft = (3 × 4 × 5) × (ft × ft × ft) = 60 cu ft. This is a volume of a rectangular prism (box). The units for volume are cubed units.

c) 80 mph × 5 hr = (80 × 5) × (mph × hr) = (80 × 5) × $(\frac{\text{mi}}{\text{hr}} \times \frac{\text{hr}}{1})$
= 400 mi

d) 54 lb ÷ 1.8 in^2 = (54 ÷ 1.8) × (lb ÷ in^2) = 30 $\frac{\text{lb}}{\text{in}^2}$ = 30 psi.

e) 56 sq ft ÷ 8 ft = (56 ÷ 8) × (sq ft ÷ ft) = 7 $\frac{\text{ft}^2}{\text{ft}}$ = 7 ft.

f) 105 m ÷ 35 m/sec = (105 ÷ 35) × (m ÷ $\frac{\text{m}}{\text{sec}}$) = 3 $\frac{\text{m}}{1} \bullet \frac{\text{sec}}{\text{m}}$
= 3 sec.

In multiplying and dividing measurements, the answer cannot be more accurate than the least accurate measurement in the product and/or quotient. We always round the answer in the product and/or quotient to same number of significant digits of the least accurate number in the problem.

Simplify the following:

Ex. 5a	9.1 ft × 8.7 ft	Ex. 5b	1.23 m × 2.4 m × 30 m
Ex. 5c	72.65 mph × 2.5 hr	Ex. 5d	5423 ton ÷ 2.30 ft^2
Ex. 5e	800 mi ÷ 4.56 hr	Ex. 5f	$2\overline{00}$ km ÷ 35 km/sec

Solution:

a) 9.1 ft × 8.7 ft = (9.1 × 8.7) × (ft × ft) = 79.17 sq ft. Round to two significant digits: 79.17 sq ft ≈ 79 sq ft.

b) 1.23 m × 2.4 m × 30 m = (1.23 × 2.4 × 30) × (m × m × m) = 88.56 cu m. Round to one significant digit:
88.56 cu m ≈ 90 cu m.

c) $72.65 \text{ mph} \times 2.5 \text{ hr} = (72.65 \times 2.5) \times (\text{mph} \times \text{hr})$
$= (72.65 \times 2.5) \times (\frac{\text{mi}}{\text{hr}} \times \frac{\text{hr}}{1}) = 181.625$ mi. Round to two significant digits: 181.625 mi ≈ 180 mi.

d) $5423 \text{ ton} \div 2.30 \text{ ft}^2 = (5423 \div 2.30) \times (\text{ton} \div \text{ft}^2)$
$= 2357.826\ldots \frac{\text{ton}}{\text{ft}^2}$. Round to three significant digits:
$2357.826\ldots \frac{\text{ton}}{\text{ft}^2} = 2360 \text{ ton / ft}^2$.

e) $800 \text{ mi} \div 4.56 \text{ hr} = (800 \div 4.56) \times (\text{mi} \div \text{hr}) = 175.438 \ldots \frac{\text{mi}}{\text{hr}}$.
Round to one significant digit: $175.438\ldots \frac{\text{mi}}{\text{hr}} \approx 200$ mph.

f) $2\overline{00} \text{ km} \div 35 \text{ km/sec} = (2\overline{00} \div 35) \times (\text{km} \div \frac{\text{km}}{\text{sec}})$
$= 5.71428\ldots \frac{\text{km}}{1} \bullet \frac{\text{sec}}{\text{km}} = 5.71428\ldots$ sec. Round to two significant digits: 5.71428… sec. ≈ 5.7 sec.

Objective 6: Rounding to the Nearest Fraction.

Some measurements need to be rounded to the nearest fraction such as to the nearest eighth, sixteenth, or thirty-second. If we are given a decimal, we rewrite the decimal over one and multiply top and bottom of the result by the number we need to have in the denominator. We then round the numerator of the answer to the nearest whole number.

Round to the indicated amount and find the error:

Ex. 6a 0.282 in to the nearest 16^{th} of an inch.
Ex. 6b 9.768 ft to the nearest 12^{th} of a foot
Ex. 6c 7.92 in to the nearest 32^{nd} of an inch.

Solution:

a) $0.282 \text{ in} = \frac{0.282}{1} \bullet \frac{16}{16} \text{ in} = \frac{4.512}{16} \text{ in} \approx \frac{5}{16}$ in.
The error is $\frac{5}{16} - 0.282 = 0.0305$ in.

b) $9.768 \text{ ft} = \frac{9.768}{1} \bullet \frac{12}{12} \text{ ft} = \frac{117.216}{12} \text{ ft} \approx \frac{117}{12} \text{ ft} = 9\frac{3}{4}$ ft
The error is $9.768 - 9\frac{3}{4} = 0.018$ ft

c) $7.92 \text{ in} = \frac{7.92}{1} \bullet \frac{32}{32} \text{ in} = \frac{253.44}{32} \text{ in} \approx \frac{253}{32} \text{ in} = 7\frac{29}{32}$ in
The error is $7.92 - 7\frac{29}{32} = 0.01375$ in.