Sect 4.5 – Special Applications of Percent Calculations

Objective 1: Understanding Percent Change.

In many situations, it is important to find the percent increase or decrease of a given quantity. The increase or decrease is always a percent of the original value. Thus, our proportion would look like this:

$$\frac{\text{increase or decrease}}{\text{original value}} = \frac{\text{percent change}}{100}$$

Some examples of percent change include:

1) Discount:
   The discount is a percent of the original value (list price).
2) Sales Tax:
   Sales Tax is a percent of the original value (price).
3) Commission
   Commission is percent of the sales.

Solve the following:

Ex. 1 A basketball auditorium increased its 19,000 seating capacity by 18%. How many seats were added to the auditorium?

Solution:

We first fill in our simple sentence:

Seats Added is 18% of the 19000.

So, we are looking for the amount:

$$\frac{A}{19000} = \frac{18}{100} \quad \text{(cross multiply)}$$

$A \bullet 100 = 19000 \bullet 18$ (simplify)

$100A = 342000$ (divide by 100)

$A = 3420.$

So, 3420 seats were added.

Ex. 2 A computer system that sold for $2,400 one year ago can now be bought for $1,800. What percent decrease does this represent?

Solution:

We first need to compute the change: $2400 – $1800 = $600

Now, we fill in our simple sentence:

$600 is a Percent of the $2400.

So we are looking for the percent:

$$\frac{600}{2400} = \frac{p}{100} \quad \text{(cross multiply)}$$

$600 \bullet 100 = 2400 \bullet p$ (simplify)
$2400p = 60000$ (divide by 2400)
$p = 25\%$

The percent decrease was 25%.

Ex. 3 A tennis racket that regularly sells for $96 is on sale for 25% off the regular price. Find the sale price.

<u>Solution:</u>

We first fill in our simple sentence:

<u> Discount </u> is <u> 25% </u> of the <u> $96 </u>.

So, we are looking for the amount:

$$\frac{A}{96} = \frac{25}{100} \quad \text{(cross multiply)}$$

$A \bullet 100 = 96 \bullet 25$
$100A = 2400$ (divide by 100)
$A = 24$. But, $96 – $24 = $72.

The sale price is $72.

But there is another way to do this problem; if you get a 25% discount, then you will pay 100% – 25% = 75%. So, we can calculate the sale price directly using the simple sentence:

<u> Sale Price </u> is <u> 75% </u> of the <u> $96 </u>.

So, in this problem, the base is 96, the percent is 75%, and we are looking for the amount:

$$\frac{A}{96} = \frac{75}{100} \quad \text{(cross multiply)}$$

$A \bullet 100 = 96 \bullet 75$ (simplify)
$100A = 7200$ (divide by 100)
$A = 72$.

So, the sale price is $72.

Ex. 4 The Coyote (carniverious eatti) needs $2880 to buy a pair of Acme® Super Jet Powered Roller Skates to catch the Road Runner (incredious Superious Speeddi). If he decides to withdraw some money from his IRA where he will receive a 10% penalty for early withdraw, how much should he withdraw so that he gets $2,880 after the penalty?

Solution:
If we try to fill in our simple sentence, we run into problems:

Penalty is 10% of the Withdraw.

The $2880 is not the amount of withdraw, but the amount the Coyote takes home after the penalty. The problem is we have two unknowns in this situation. We need to approach this problem another way. If 10% of the withdraw is taken for the penalty, then the coyote will receive 100% – 10% = 90% of the withdraw. Then,

$2880 is 90% of the Withdraw.

So, we are looking for the base:

$$\frac{2880}{B} = \frac{90}{100} \quad \text{(cross multiply)}$$

$$2880 \bullet 100 = B \bullet 90 \quad \text{(simplify)}$$

$$288000 = 90B \quad \text{(divide by 90)}$$

$$B = 3200.$$

He should withdraw $3200.

Ex. 5 If Juan paid $31.50 in sales tax on a $400 stereo system, what was the sales tax rate?

Solution:
We first fill in our simple sentence:

$31.50 is a Percent of the $400.

So, we are looking for the percent:

$$\frac{31.50}{400} = \frac{p}{100} \quad \text{(cross multiply)}$$

$$31.50 \bullet 100 = 400 \bullet p$$

$$400p = 3150 \quad \text{(divide by 400)}$$

$$p = 7.875\%.$$

The sales tax rate is 7.875%.

Ex. 6 If Joe received a 4.5% commission on the sale of $180,000 home in Canyon Lake, how much was his commission?

Solution:
We first fill in our simple sentence:

Commission is 4.5% of the $180000.

So we are looking for the amount:

$$\frac{A}{180000} = \frac{4.5}{100} \quad \text{(cross multiply)}$$

$$A \bullet 100 = 180000 \bullet 4.5 \quad \text{(simplify)}$$

$$100A = 810000 \quad \text{(divide by 100)}$$

$A = 8100.$

So, his commission was \$8,100.

Ex. 7 During a malfunction, the normal voltage, 120 volts, drops by 30%. What is the reduced voltage?

<u>Solution:</u>

If the voltage was reduced by 30%, then the voltage level is 100% – 30% = 70% of the original level.

Filling in our simple sentence:

<u> Reduced Voltage </u> is <u> 70% </u> of the <u> 120 volts </u>.

So we are looking for the amount:

$$\frac{A}{120} = \frac{70}{100} \quad \text{(cross multiply)}$$

$$A \bullet 100 = 120 \bullet 70$$

$$100A = 8400 \quad \text{(divide by 100)}$$

$$A = 84.$$

So, the reduced voltage was 84 volts.

Ex. 8 For Katz Catering Service, the cost of produce has risen 35% over the last year. If a five-pound bag of oranges now costs \$5.67, how much did it cost last year?

<u>Solution:</u>

If we try to fill in our simple sentence, we run into the same problem as before:

<u> Increase </u> is <u> 35% </u> of the <u> Last Year's Value </u>.

Again, we need to approach this problem from another angle. If Last Year's Value represents 100% and the value increased by 35%, then this year's value is 100% + 35% = 135% of last year's value.

So, our simple sentence becomes:

<u> \$5.67 </u> is <u> 135% </u> of the <u> Last Year's Value </u>.

So, we are looking for the base:

$$\frac{5.67}{B} = \frac{135}{100} \quad \text{(cross multiply)}$$

$$5.67 \bullet 100 = B \bullet 135 \quad \text{(simplify)}$$

$$135B = 567 \quad \text{(divide by 135)}$$

$$B = 4.2.$$

A five-pound bag of oranges was \$4.20 last year.

Objective 2: Understanding Interest.

When a person gets a loan from the bank, that person has to pay back the amount of the loan plus interest. The interest is the fee that the person has to pay for using the money from the bank. In the same way, if a person opens a savings account, the bank pays that person interest since the bank gets to use that person's money while it is in the account. The Interest is the fee paid for borrowing money. The amount of the loan or the amount in a savings account is called the principal. The interest rate is the rate that interest is charged on the amount borrowed.

The annual interest is a percentage of the principal:

$$\frac{\text{annual interest}}{\text{principal}} = \frac{\text{annual interest rate}}{100}$$

The total interest = annual interest $\times$ time in years

<u>Solve the following:</u>

Ex. 9 A manufacturer gets 9-month loan of \$12,000 at 7½% annual interest. Find the total the manufacturer had to pay back.

<u>Solution:</u>

Since the annual interest is a percent of the principal, then:

<u>The annual interest</u> is 7½% of \$12,000.

So we are looking for the amount:

$$\frac{A}{12000} = \frac{7\frac{1}{2}}{100} \quad \text{(cross multiply)}$$

$A \bullet 100 = 12000 \bullet (7\frac{1}{2})$

$100A = 90000$ (divide by 100)

$A = 900.$

Since 9 months is $\frac{9}{12}$ or $\frac{3}{4}$ of year, then the total interest is

$900 \times \frac{3}{4} = \$675$

The total to be paid back is principal plus interest:

12000 + 675 = 12675.

The manufacturer had to pay \$12,675 on the loan.

Objective 3: Understanding Efficiency.

The efficiency of a machine and/or process compares the energy output to the energy input. Thus, the output energy is a percentage (efficiency) of the input energy.

Solve the following:

Ex. 10 An engine that is rated at 220 hp only delivers 187 hp to the transmission. What is the engine's efficiency?

Solution:

The output energy (187 hp) is a percent of the input energy (220 hp):
Thus, 187 is P% of 220.
So, we are looking for the percent:

$$\frac{187}{220} = \frac{p}{100} \quad \text{(cross multiply)}$$

$187 \bullet 100 = 220 \bullet p$

$220p = 18700$ (divide by 220)

$p = 85\%$.

The efficiency rate is 85%.

Ex. 11 An engine with an efficiency of 75% has an output energy of 124.5 hp. What is the input energy?

Solution:

The output energy (124.5 hp) is 75% of the input energy:
Thus, 124.5 is 75% of the input energy.
So, we are looking for the base:

$$\frac{124.5}{B} = \frac{75}{100} \quad \text{(cross multiply)}$$

$124.5 \bullet 100 = 75 \bullet B$

$75B = 12450$ (divide by 75)

$B = 166$

The input energy was 166 hp.

Objective 4: Understanding Tolerance.

In designing a part, **Tolerance** is the allowed error. In other words, if a bolt is designed to be 2.125 ± 0.025 inches long, this means the actual length of bolt has to be within 0.025 inches of 2.125 inches. The longest the bolt can be is 2.125 + 0.025 in. = 2.15 in. and the shortest it can be 2.125 – 0.025 in. = 2.1 in. These values, 2.15 in. & 2.1 in., are the tolerance limits. The tolerance can also be written as a percent.

The tolerance is a percent (tolerance) of the measurement.

In the example above, 0.025 is a percent of 2.125. Thus,

$$\frac{0.025}{2.125} = \frac{P}{100}$$ (cross multiply)

$0.025 \bullet 100 = 2.125P$

$2.125P = 2.5$ (divide by 2.125)

$P = 1.1764\ldots \approx 1.2\%$

The percent tolerance is ≈ 1.2%.

Solve the following:

Ex. 12 The resistance of a particular resistor needs to be 3400 ± 5% ohms. Find the tolerance in ohms and the tolerance limits.

Solution:

Since the tolerance is 5% of the measurement (3400 ohms), then

$$\frac{A}{3400} = \frac{5}{100}$$ (cross multiply)

$A \bullet 100 = 3400(5)$

$100A = 17000$ (divide by 100)

$A = 170$ ohms

The limits are 3400 – 170 = 3230 and 3400 + 170 = 3570.

Thus, the tolerance is 170 ohms and the limits are 3230 ohms to 3570 ohms.