Sect 4.2 – Applications of Ratios and Proportions

Objective 1: Scale Drawings

In designing blueprints for a house or creating a model airplane, the measurements on the blueprints or on the model airplane must be scaled down in the same proportion to match the object in real life accurately. The purpose is to represent the object in real life, but yet to be small enough to fit on a piece of paper or, in the case of a model airplane, to fit it into a display cabinet. The ratio of a length in real life to the length on the model or blueprint is called the **scale factor** of the reduction.

Solve the following:

Ex. 1 A rectangular room measures 3.5 inches long by 6.5 inches wide on a blueprint. If the actual length of the room is 14 feet, a) find the width of the room. b) Also find the scale factor of the reduction.

Solution:

a) $\frac{\text{blueprint}}{\text{real life}}: \quad \frac{3.5\,\text{in}}{14\,\text{ft}} = \frac{6.5\,\text{in}}{w}$

Convert the fourteen feet into inches: 14(12) = 168 in

$\frac{3.5\,\text{in}}{168\,\text{in}} = \frac{6.5\,\text{in}}{w}$ (cross multiply)

$3.5w = 168(6.5)$

$\frac{3.5w}{3.5} = \frac{1092}{3.5}$ (divide by 3.5)

$w = 312$ in

But, 312 ÷ 12 = 26 ft.

So, the actual width of the room is 26 feet.

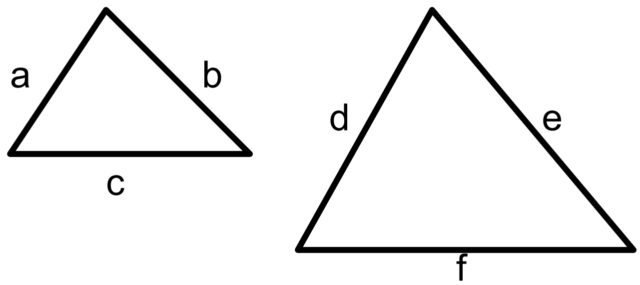
b) Since 3.5 inches corresponds to 168 inches in real life, then the scale factor is the ratio of 168 to 3.5:

$$\frac{168}{3.5} = \frac{168 \div 3.5}{1} = \frac{48}{1}.$$

So the scale of the reduction is 48 to 1.

Objective 2: Similar Figures.

Two polygons are **<u>similar</u>** if they have the same shape, but not necessarily the same size. The ratios of corresponding sides of similar polygons are equal. In other words, if the two triangles pictured are similar, then the following is true:



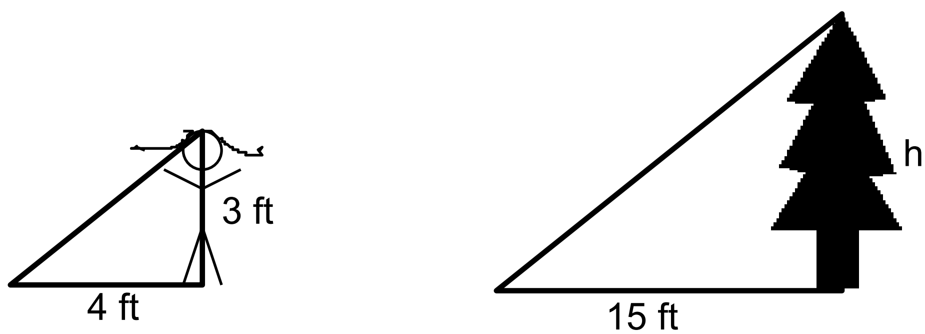
$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

<u>Solve the following:</u>

Ex. 2 A tree casts a 15-foot shadow at the same time a child 3 feet tall casts a 4-foot shadow. How tall is the tree?

<u>Solution:</u>

We begin by drawing a picture:



These two triangles are similar triangles so we will take the ratio the lengths of the shadows and set it equal to the ratio of the heights:

$$\frac{\text{Length of the child's shadow}}{\text{Length of the tree's shadow}} = \frac{\text{Child's height}}{\text{Tree's height}}$$

$\frac{4}{15} = \frac{3}{h}$ Cross multiply and solve:

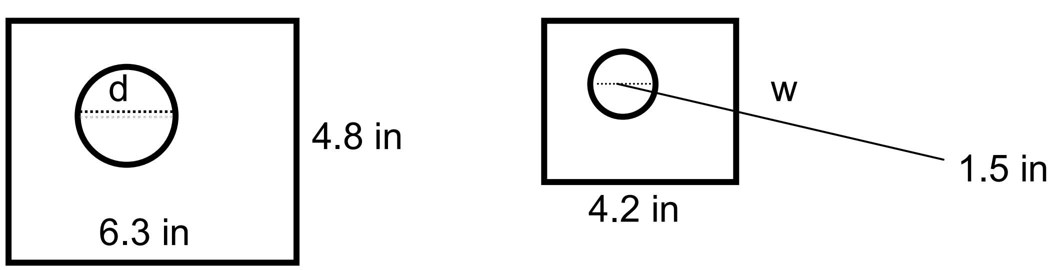
$4h = 15 \bullet 3$

$\frac{4h}{4} = \frac{45}{4}$

$h = 11.25$ ft

The tree is 11.25 feet tall.

Ex. 3 If the following figures are similar, find the missing dimensions.



Solution:
Since we know the lengths of both figures, we will use them to form our first ratio. Let's first find the width of the smaller rectangle

$$\frac{\text{larger figure}}{\text{smaller figure}}: \qquad \frac{6.3}{4.2} = \frac{4.8}{w} \quad \text{(cross multiply)}$$

$$6.3w = 4.2(4.8)$$
$$6.3w = 20.16 \qquad \text{(divide by 6.3)}$$

$$w = 3.2 \text{ in}$$

So, the width of the smaller figure is 3.2 in.
Now, let's get the diameter of the larger circle:

$$\frac{\text{larger figure}}{\text{smaller figure}}: \qquad \frac{6.3}{4.2} = \frac{d}{1.5} \quad \text{(cross multiply)}$$

$$6.3(1.5) = 4.2d$$
$$9.45 = 4.2d \qquad \text{(divide by 4.2)}$$

$$d = 2.25 \text{ in}$$

The diameter of the larger circle is 2.25 in.

Objective 3: Direct Proportions

In general, there are two different types of proportions that can be used to solve many types of applications. The first type is called a **direct proportion**. Two quantities are directly proportional if an increase in one quantity leads to a proportional increase in the other quantity (i.e., if the first quantity is tripled, then the second quantity is tripled). For example, if your speed is fixed, then the distance traveled is directly proportional to time driving on the road; the longer a person drives at the same speed, the more distance the person has traveled.

Solve the following:

Ex. 4 The resistance of a wire is directly proportional to its length. If a 2-ft wire has a resistance of 1.2 ohms, find the length of the same type of wire that will have a resistance of 4.5 ohms.

Solution:
Since the resistance is directly proportional to the length, we expect that as the resistance increases, the length of the wire will also increase. So, as a check for solving this problem, since the resistance is higher in the length of the piece we are solving than for the piece

we are given, our answer should be bigger than the length of the piece we are given.

$$\frac{\text{shorter piece}}{\text{longer piece}}: \qquad \frac{2\text{ ft}}{L} = \frac{1.2\text{ohms}}{4.5\text{ohms}} \qquad \text{(drop the units)}$$

$$\frac{2}{L} = \frac{1.2}{4.5} \qquad \text{(cross multiply)}$$

$2(4.5) = 1.2L$

$9 = 1.2L$ (divide by 1.2)

$L = 7.5$ ft Notice this is larger than 2 ft length we were given. The length of the wire is 7.5 feet.

Ex. 5 A punch recipe calls for $1\frac{1}{2}$ gallons of sherbet for every 5 liters of 7-up. How many gallons of sherbet should be used with 2 liters of 7-up?

Solution:

This is a direct proportion since as the amount of 7-up increases, the amount of sherbet also increases by the same proportion. Since the amount of 7-up is smaller in the part we are solving than in what we are given, then the amount of sherbet has to be smaller than $1\frac{1}{2}$ gal.

$$\frac{\text{smaller amount}}{\text{larger amount}}: \qquad \frac{s}{1\frac{1}{2}\text{gal}} = \frac{2L}{5L} \quad \text{(drop the units and cross multiply)}$$

$\left(1\frac{1}{2}\right)(2) = 5s$

$3 = 5s$ (divide by 5)

$s = 0.6$ gallons which is less than $1\frac{1}{2}$ gallons.

So, 0.6 gallons of sherbet is needed.

Ex. 6 In a money market account, if \$1500 earned \$82.50 in interest during a given period of time, how much interest would \$3300 earned in the same money market account during the same period of time?

Solution:

This is direct proportion since the more money one has invested, the more interest one earns. Since the amount of money that we are finding the interest on is larger than the amount of money with the given interest, the answer should be larger than \$82.50:

$$\frac{\text{smaller amount}}{\text{larger amount}}: \qquad \frac{\$82.50}{i} = \frac{\$1500}{\$3300} \qquad \text{(cross multiply)}$$

82.50(3300) = 1500 i
272250 = 1500 i (divide by 1500)
i = $181.50 which is larger than $82.50.

The interest was $181.50.

Objective 4: Inverse Proportions

The second type is called an **inverse proportion**. Two quantities are inversely proportional if an increase in one quantity leads to a proportional ***decrease*** in the other quantity (i.e., if the first quantity is doubled, then the second quantity is halved). For example, if your distance is fixed, then the time traveled is inversely proportional to your driving speed; the faster a person drives, the less time it takes to reach one's destination. When we set-up a inverse proportion, we will need to invert one of the ratios to put the equation in balance so that ratios will either both increase or both decrease as we change a quantity.

Ex. 7 Driving at 42 mph, a trip takes 3 hours to complete. If the speed is increased to 56 mph, how long will the trip take?

Solution:

This is an inverse proportion since as the speed increases, the time of the trip decreases by a proportional amount. Since the speed is increasing, the time should decrease, so our answer will be less than 3 hours.

To set-up the first ratio, put the slower speed over the faster: $\frac{42\text{mph}}{56\text{mph}}$

We will then have to invert the time ratio, by putting the shorter time over the longer time: $\frac{t}{3}$

Thus, our proportion is: $\frac{42\text{mph}}{56\text{mph}} = \frac{t}{3}$ (cross multiply)

42(3) = 56t
126 = 56t (divide by 56)
t = 2.25 which is smaller than 3

So, it will take 2 hr 15 min to make the trip.

Ex. 8 The electrical resistance of a wire with a given length is inversely proportional to square of the diameter of the wire. If the wire with a diameter of 82.48 mils has a resistance of 4.5 ohms, what is the resistance (to the nearest tenth) of the same wire with a diameter of 62.8 mils.

Solution:

This is an inverse proportional. Since the diameter is decreasing, then the square of the diameter is decreasing, then the resistance is increasing. So, our answer should be greater than 4.5 ohms.

Write the ratio of the square of the smaller diameter over the square of the larger diameter: $\frac{(62.8\,\text{mils})^2}{(82.48\,\text{mils})^2}$

Now, we will need to invert the ratio of the resistances by putting the smaller resistance over the larger resistance: $\frac{4.5\,\text{ohms}}{r}$

Thus, our proportion is: $\frac{(62.8\,\text{mils})^2}{(82.48\,\text{mils})^2} = \frac{4.5\,\text{ohms}}{r}$ (simplify)

$\frac{3943.84}{6802.9504} = \frac{4.5}{r}$ (cross multiply)

3943.84r = 6802.9504(4.5)

3943.84r = 30613.2768 (divide by 3943.84)

r = 7.7623… ≈ 7.8 ohm which is greater than 4.5 ohms

Hence, the resistance will be ≈ 7.8 ohms.

With gears and pulleys, the size of the gear or pulley is inversely proportional to the speed of the gear or pulley. With gears, the gear speed is measured in revolutions per minute (rpm) and the size of the gear is measured in the number of teeth it has. For pulleys, the pulley speed is also measured in revolutions per minute (rpm), but the size of the pulley is measured by its diameter.

Gears

$$\frac{\text{speed of gear a}}{\text{speed of gear b}} = \frac{\text{teeth in gear b}}{\text{teeth in gear a}}$$

Pulleys

$$\frac{\text{speed of pulley a}}{\text{speed of pulley b}} = \frac{\text{diameter of pulley b}}{\text{diameter of pulley a}}$$

When dealing with gears in a car, then

$$\frac{\text{drive shaft speed}}{\text{rear axle speed}} = \frac{\text{teeth in ring gear on the axle}}{\text{teeth in pinion gear on the drive shaft}}$$

Solve the following:

Ex. 9 If the pinion gear has 12 teeth and the ring gear has 45 teeth, what is the rear axle speed when the drive shaft turns at 2700 rpm?

Solution:

Plug the numbers into:

$$\frac{\text{drive shaft speed}}{\text{rear axle speed}} = \frac{\text{teeth in ring gear on the axle}}{\text{teeth in pinion gear on the drive shaft}}$$

$\frac{2700\,rpm}{r} = \frac{45\,teeth}{12\,teeth}$ (cross multiply)
2700(12) = 45r
32400 = 45r (divide by 45)
r = 720
The rear axle speed is 720 rpm.

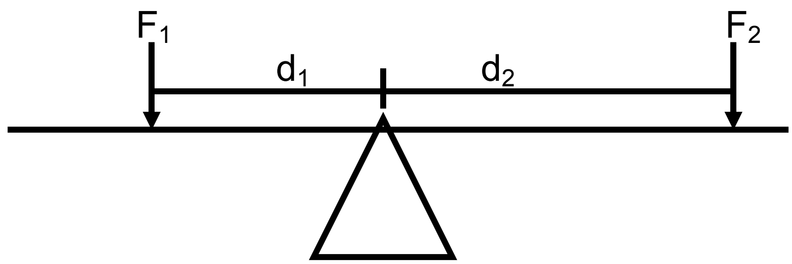
Ex. 10 If driving gear with 20 teeth is turned by a shaft at 60 rpm, how many teeth will the driven gear have if it turns at 80 rpm?
Solution:
Let gear a be the driving gear. The speed is inversely proportional to the number of teeth. Since the driven gear b is going faster, it has to have less teeth.
$\frac{60\,rpm}{80\,rpm} = \frac{t}{20\,teeth}$ (cross multiply)
60(20) = 80t
1200 = 80t (divide by 80)
15 = t
So, the driven gear has 15 teeth.

Ex. 11 If a pulley with diameter of 16 inches rotates at 360 rpm, what speed is a pulley with a diameter of 20 inches rotating?
Solution:
The speed of a pulley is inversely proportional to its diameter. Since the second pulley is larger, its speed should be slower.
$\frac{16}{20} = \frac{r}{360}$ (cross multiply)
16(360) = 20r
5760 = 20r (divide by 20)
288 = r
It is rotating at 288 rpm.

Ex. 12 For gases at a fixed temperature, the pressure is inversely proportional to the volume. If 24 cu ft of air at 10 psi is compressed to 8 cu ft, what is the new pressure?
Solution:
As the volume decreases, the pressure increases. So the new pressure should be higher than the old pressure.
$\frac{24\,cu\,ft}{8\,cu\,ft} = \frac{p}{10\,psi}$ (cross multiply)
24(10) = 8p

$240 = 8p$ (divide by 8)
$p = 30$ psi
The new pressure is 30 psi.

The force on the end of a lever arm is inversely proportional to the distance a lever is from the pivot point. If you think of a seesaw, the heavier a person is compare to the person on the other end, the further that person needs to be from the pivot point to balance the seesaw.



Ex. 13 A crowbar 27 inches long is pivoted 3 inches from the end. If 100-lb of force is applied to the long end, how many pounds of force is there to lift an object at the short end?

Solution:

This is essentially a lever-fulcrum problem as described above. Since the force is inversely proportional to the distance from the pivot point, as the distance from the pivot point gets smaller, the force gets larger. The length of the longer end from the pivot point is $27 - 3 = 24$ in.

$\frac{3\,\text{in}}{24\,\text{in}} = \frac{100\text{–lb}}{f}$ (cross multiply)
$3f = 24(100)$
$3f = 2400$ (divide by 3)
$f = 800$ lb

The amount of force exerted to lift an object is 800 lb.