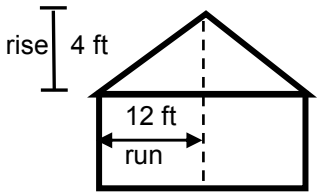
Sect 4.1 – Ratios and Proportions

Objective 1: Understanding Ratios

A **ratio** is a comparison of two quantities that have the same type of units. If we want to write the ratio of 5 to 7, we have three different ways to express it. We can write "5:7," or "5 to 7," or "$\frac{5}{7}$." All three forms are equivalent and no matter which form we use, we will express our ratios using whole numbers. Also, since we are comparing two quantities with the same type of units, if the units are not already the same, we will convert them so that they are the same and then leave them off in our answer. Some examples of ratios include roof pitch, gear ratios, pulley ratios, and compression ratios:

Pitch = $\frac{\text{rise}}{\text{run}}$

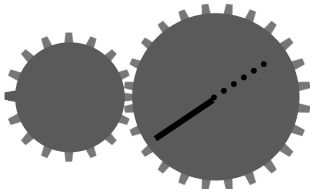
= $\frac{4}{12}$ or 4 : 12



(Usually expressed with a denominator of 12).

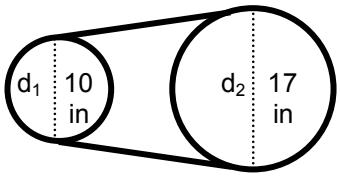
Gear Ratio = $\frac{\text{number of teeth on the driving gear}}{\text{number of teeth on the driven gear}}$

= $\frac{24}{16}$ = $\frac{3}{2}$ or 3 : 2 or 1.5 : 1



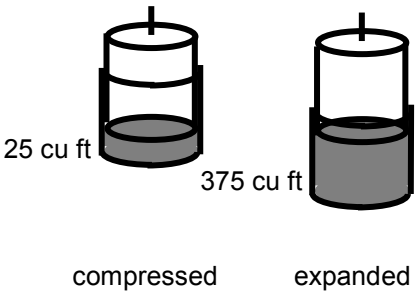
Pulley Ratio = $\frac{\text{diameter of one pulley } (d_1)}{\text{diameter of a second pulley } (d_2)}$

= $\frac{10}{17}$ or 10 : 17



Compression Ratio = $\frac{\text{expanded volume}}{\text{compressed volume}}$

= $\frac{375}{25}$ = $\frac{15}{1}$ or 15 : 1



Solve the following:

Ex. 1 If the compressed volume of a cylinder of an engine is 12.8 cu in. and the expanded volume is 56 cu in., find the compression ratio.
Solution:

Compression Ratio = $\frac{\text{expanded volume}}{\text{compressed volume}} = \frac{56}{12.8} = \frac{4.375}{1}$.

The compression ratio is 4.375 **:** 1.

Ex. 2 If the diameter of the first pulley is 16 cm and the diameter of the second pulley is 32 cm, find the pulley ratio.
Solution:

Pulley Ratio = $\frac{\text{diameter of one pulley } (d_1)}{\text{diameter of a second pulley } (d_2)} = \frac{16}{32} = \frac{1}{2}$.

The pulley ratio is 1 **:** 2.

Ex. 3 Find the rear-axle ratio of a car if the ring (driving) gear has 42 teeth and the pinion (driven) gear has 12 teeth.
Solution:

Gear Ratio = $\frac{\text{number of teeth on the driving gear}}{\text{number of teeth on the driven gear}} = \frac{42}{12} = \frac{7}{2}$

The rear-axle ratio is 7 **:** 2.

Objective 2: Solving Simple Equations.

In Algebra, when there is a number that we do not know its value, we represent the number using a letter like x. So, if we want to write five times an unknown number, we write 5•x or 5x. Using this idea, we can use what are called equations to help us find these unknown numbers for a particular example. Let's state a definition.

An **equation** is a statement that two quantities are equal. An equation can be as simple as 4 quarters = $1, or it could be more complex like $3x = 2.7$ and $2x^2 - 5x + 4 = 23$. A **solution** to an equation is the value of x that makes the equation true. For example, x = 0.9 is a solution to the equation 3x = 2.7 since if we replace x by 0.9 and do the multiplication, we get: 3(0.9) = 2.7.

Solve the following:

Ex. 4 $4x = 28$

Solution:
Since $4 \bullet 7 = 28$, then $x = 7$ has to be the solution. We can also get the solution by dividing 28 by 4 since $28 \div 4 = 7$.

The property the allows us to solve this equation is called the multiplication property of equality. It says we can multiply or divide both sides by the same non-zero number. In our example, we are dividing both sides by 4 to solve for x:

Multiplication Property of Equality:
If $A = B$ and $C \neq 0$, then $A \bullet C = B \bullet C$ and $\frac{A}{C} = \frac{B}{C}$ are equivalent equations to $A = B$ (i.e., they have the same solutions)

Solve the following:

Ex. 5a $-2.4x = 9.84$

Solution:
We will divide both sides by the number in front of x:

$$\frac{-2.4x}{-2.4} = \frac{9.84}{-2.4}$$

$$x = 9.84 \div (-2.4)$$

$$x = -4.1$$

Ex. 5b $\frac{3}{4}x = \frac{5}{6}$

Solution:
We will divide both sides by the number in front of x:

$$\frac{\frac{3}{4}x}{\frac{3}{4}} = \frac{\frac{5}{6}}{\frac{3}{4}}$$

$$x = \frac{5}{6} \div \frac{3}{4}$$

$$x = \frac{10}{9}$$

Objective 3: Understanding Proportions.

A **proportion** is a statement that two ratios are equal.

A proportion takes a form that is similar to an analogy in English class. For example, a puppy is to a dog as a kitten is to a cat is an analogy, but we can write this as a proportion. A puppy is to a dog would be a fraction on one side of the equal sign and a kitten is to a cat would be a fraction on the other side:

$$\frac{\text{puppy}}{\text{dog}} = \frac{\text{kitten}}{\text{cat}}$$

Write the following as a proportion:

Ex. 6 Fifty is to 35 as thirty is to 21

Solution:

Fifty is to 35: $\frac{50}{35}$

thirty is to 21: $\frac{30}{21}$

So, the proportion is $\frac{50}{35} = \frac{30}{21}$.

Notice that $\frac{50}{35}$ reduces to $\frac{10}{7}$ and $\frac{30}{21}$ also reduces to $\frac{10}{7}$. Another way to verify that a proportion is true is the multiply the denominator of the first ratio with the numerator of the second ratio: $\frac{50}{35} = \frac{30}{21} \Rightarrow 35(30) = 1050$

Next, multiply the numerator of the first fraction with the denominator of the second: $\frac{50}{35} = \frac{30}{21} \Rightarrow 50(21) = 1050$

If the two results are the same, then the proportion is true. This technique is called cross-multiplication.

Cross Multiplication

For $b \neq 0$ and $d \neq 0$, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$. "ad" and "bc" are called the cross products.

Now, we will examine how to solve for a missing number in a proportion. First, we will cross multiply. Next, we will simplify each cross product. Finally, we will use the multiplication property of equality to solve for the unknown number. Let's try some examples.

Solve:

Ex. 7 $\frac{n}{8} = \frac{7}{11}$

Solution:

$\frac{n}{8} = \frac{7}{11}$ (cross multiply)

$n \bullet 11 = 8 \bullet 7$ (simplify)

$11n = 56$ (divide by 11)

$\frac{11n}{11} = \frac{56}{11}$

$n = \frac{56}{11}$ or $5\frac{1}{11}$ or $5.\overline{09}$.

Ex. 8 $\frac{0.8}{11} = \frac{9.5}{n}$

Solution:

$\frac{0.8}{11} = \frac{9.5}{n}$ (cross multiply)

$0.8 \bullet n = 11 \bullet 9.5$ (simplify)

$0.8n = 104.5$ (divide by 0.8)

$\frac{0.8n}{0.8} = \frac{104.5}{0.8}$

$n = 104.5 \div 0.8 = 130.625$.

Ex. 9 $\frac{3\frac{1}{4}\text{ in}}{6\frac{3}{11}\text{ ft}} = \frac{n}{5\frac{4}{13}\text{ ft}}$

Solution:

Since n is on top, then the answer will be in inches.

$\frac{3\frac{1}{4}}{6\frac{3}{11}} = \frac{n}{5\frac{4}{13}}$ (cross multiply)

$\left(3\frac{1}{4}\right)\left(5\frac{4}{13}\right) = \left(6\frac{3}{11}\right)n$ (multiply the mixed numbers on the left)

$\frac{17\frac{1}{4}}{6\frac{3}{11}} = \frac{6\frac{3}{11}n}{6\frac{3}{11}}$ (divide by $6\frac{3}{11}$)

$n = 2\frac{3}{4}$ in or 2.75 in

Now, let's try some applications.

Solve the following:

Ex. 10 If 32 tapered pins can be machines from a 15-foot steel rod, how many tapered pins can be made from a 12-foot steel rod?

Solution:

Write pins over steel rod:

$\frac{\text{pins}}{\text{steel rod}}$: $\frac{32}{15} = \frac{p}{12}$ (cross multiply)

$32(12) = 15p$

$\frac{384}{15} = \frac{15p}{15}$ (divide by 15)

$p = 25.6$ (we need to round down to 25 pins)

Twenty-five tapered pins can be made from a 12-foot rod.

Ex. 11 A rectangular box shaped deep fryer 2 ft deep holds 3 gallons of cooking oil when it is full. How much cooking oil is left in the deep fryer when the depth of the oil is $1\frac{1}{3}$ ft?

Write full over part full:

$\frac{\text{full}}{\text{part full}}$: $\frac{2}{1\frac{1}{3}} = \frac{3}{g}$ (cross multiply)

$2g = \left(1\frac{1}{3}\right)(3)$

$\frac{2g}{2} = \frac{4}{2}$ (divide by 2)

$g = 2$

There are 2 gallons of oil.

Ex. 12 A roof has a pitch of 5 **:** 12. If the roof has a run of 16 ft 6 in, find the rise.

First convert 16 ft 6 in into feet:

Since 6 in = $\frac{1}{2}$ ft, then 16 ft 6 in = $16\frac{1}{2}$ ft

Pitch = $\frac{\text{rise}}{\text{run}} = \frac{5}{12} = \frac{r}{16\frac{1}{2}}$ (cross multiply)

$5\left(16\frac{1}{2}\right) = 12r$

$\frac{82\frac{1}{2}}{12} = \frac{12r}{12}$ (divide by 12)

$r = 6\frac{7}{8}$

The rise is $6\frac{7}{8}$ ft or 6 ft $10\frac{1}{2}$ in.