Sect 12.2 – Measures of Central Tendency

When given a set of data, we want to calculate two numbers that describe the data. The first number is a **measure of central tendency**, which is a "middle number" or average number of a set of data. The second number is a **measure of dispersion** which indicates how "spread out" the values in data are or how much variation there is between the data values. In this section, we will discuss measures of central tendency and in the next section, we will discuss measures of dispersion.

Objective #1: Understanding Mean, Median, Mode and Weighted Mean.

The **mean** is just another name for the average of a set of numbers. To find the mean, we simply add the numbers together and then divide by the number of numbers.

The **median** is the "middle number" of a set of numbers. To find the median of a set of numbers, first list the numbers from smallest to largest. If there is an odd number of numbers in the list, the median is the middle number. If there is an even number of numbers in the list, the median is the average of the two middle numbers.

Solve the following:
Ex. 1   The lengths of time of five different customer service calls
             during the last hour were 3.4 minutes, 9.7 minutes, 4.2 minutes,
             1.6 minutes, and 1.8 minutes. Find the mean and the median
             length of time of a call.
Solution:
First, list the numbers from smallest to largest:
        1.6, 1.8, 3.4, 4.2, 9.7
To find the mean, find the sum of the numbers and then divide by 5:
Mean = (1.6 + 1.8 + 3.4 + 4.2 + 9.7) ÷ 5 = 20.7 ÷ 5 = 4.14, so the
Mean ≈ 4.1 minutes.
Since there are five numbers and five is an odd number, then the median is the middle number which is 3.4, so the Median = 3.4 minutes.
Thus, the mean ≈ 4.1 minutes and the median = 3.4 minutes.

Ex. 2   The inspection time of six different plants were 9.3 hours,
             8.2 hours, 7.4 hours, 9.3 hours, 9.3 hours, and 6.1 hours. Find
             the mean and the median inspection time.
       Solution:
       First, list the numbers from smallest to largest:
             6.1, 7.4, 8.2, 9.3, 9.3, 9.3
       To find the mean, find the sum of the numbers and then divide by 6:
       Mean = (6.1 + 7.4 + 8.2 + 9.3 + 9.3 + 9.3) ÷ 6
             = 49.6 ÷ 6 = 8.26… ≈ 8.3 hours
       Since there are six numbers and six is an even number, then the
       median is the average of the two middle numbers:
       Median = (8.2 + 9.3) ÷ 2 = 17.5 ÷ 2 = 8.75 ≈ 8.8 hours.
       Hence, the mean is 8.3 hours and the median is 8.8 hours.

Now, let's define the mode.

The **mode** is the number that occurs most often in a set of numbers. If no
number appears more often than any other, then there is no mode. If there
is more than one number that appears most often, then each number is the
mode.

Solve the following:

Ex. 3   Find the mode of the data from example #1: 3.4 minutes,
             9.7 minutes, 4.2 minutes, 1.6 minutes, and 1.8 minutes.
       Solution:
       Since every number occurs only one, there is no mode.

Ex. 4   Find the mode of the data from example #2: 9.3 hours,
             8.2 hours, 7.4 hours, 9.3 hours, 9.3 hours, and 6.1 hours.
       Solution:
       Since 9.3 appears three times whereas all the other numbers appear
       once, then the mode is 9.3 hours.

Ex. 5   Find the mode of 9, 11, 4, 3, 4, 11, 8, 1, 11, 5, and 4.
       Solution:
       Since both 4 and 11 occur three times and all the other numbers
       occur once, the modes are 4 and 11.

Ex. 6   Find the mean, median and mode of following donations to a charity $6.30, $1.75, $4.26, $9, $0.72, and $1.10.

Solution:

First write in order from smallest to largest:

$0.72, $1.10, $1.75, $4.26, $6.30, $9.

Mean = (0.72 + 1.1 + 1.75 + 4.26 + 6.3 + 9) ÷ 6

= 23.13 ÷ 6 = 3.855 ≈ $3.86

Median = (1.75 + 4.26) ÷ 2 = 6.01 ÷ 2 = 3.005 ≈ $3.01

Mode = None.

Thus, the Mean ≈ $3.86, the Median = $3.01, and the Mode = None.

Ex. 7   The thickness of four wooden boards is 0.250 in, $\frac{3}{8}$ in, $\frac{1}{4}$ in, and $\frac{5}{16}$ in. Find the mean, median, and mode of thickness of the boards.

Solution:

Since $\frac{1}{4}$ = 0.25, $\frac{3}{8}$ = 0.375, and $\frac{5}{16}$ = 0.3125 are nice, terminating decimals, we can convert the fractions into decimal.

From smallest to largest: 0.25, 0.25, 0.3125, 0.375.

Mean = (0.25 + 0.25 + 0.3125 + 0.375) ÷ 4

= 1.1875 ÷ 4 = 0.296875 ≈ 0.297 in

Median = (0.25 + 0.3125) ÷ 2 = 0.5625 ÷ 2 = 0.28125 ≈ 0.281 in

Mode = 0.250 in.

Thus, the Mean = 0.297 in, the Median = 0.281 in, and the Mode = 0.25 in

Weighted Mean

If some of the data has a stronger bearing on the average than the other data, then the weight of that data must be used. Typically, one takes the data and multiplies it by its weight. Add the results and then divide by the total of the weights. Let's explore how the weighted mean is used in several examples.

Solve the following:

Ex. 8   For the fall semester, Pearl received the following grades:

Number of Credit Hours	Grade
4	A
2	B
3	C
3	B
1	A

Calculate Pearl's GPA.

Solution:

At most universities, an A is worth 4 points per credit, a B is worth 3 points per credit, a C is worth 2 points per credit, a D is worth 1 point per credit, and an F is worth 0 points per credit. We begin by multiplying the number of credits hours of each course by the grade points received for that course and add the results. Then, we divide the answer by the total number of credits:

Number of Credit Hours	Grade	Total Points
4	A = 4	4 • 4 = 16
2	B = 3	2 • 3 = 6
3	C = 2	3 • 2 = 6
3	B = 3	3 • 3 = 9
1	A = 4	1 • 4 = 4
Total: 13 credits		**Total: 41 points**

GPA = 41 points ÷ 13 credits = 3.153846 ... ≈ 3.15.

Ex. 9   For the first week of November, Juantia received the following overall ratings (on a scale of 1 to 5) from her customers who submitted a ratings card: 5, 4, 5, 3, 5, 5, 5, 4, 4, 3, 1, 5, 5, 4, 4, 5, 5, 5, 4, 5, 4, 5, 5, 4, 4, 3, 4, 5, 5, 5, 4, 3, 5, 5, 5, 4, 5, 4, 5, 5, 5, 4, 5, 5, 3, 5, & 5. Find the mean of her customers' ratings.

Solution:

        Rather than trying to add on the numbers up and dividing by the number of numbers, let's make a tally of the number of 1's, 2's, 3's, 4's, and 5's. Thus, our catagories or classes will be the 1's, 2's, 3's, 4's, and 5's. The number of tallys for each class is called the frequency of each class. We can then muliply the frequency times the

weight of each class, add the results and divide by the total of the frequencies.

Class	Tally	Frequency	Weight•Freq.
1	|	1	1(1) = 1
2		0	2(0) = 0
3	︱︱︱︱︱	5	3(5) = 15
4	︱︱︱︱ ︱︱︱︱ ︱︱︱︱	14	4(14) = 56
5	︱︱︱︱ ︱︱︱︱ ︱︱︱︱ ︱︱︱︱ ︱︱︱︱ ︱︱	27	5(27) = 135
		Sum = 47	Sum = 207

Mean = 207 ÷ 47 = 4.404... ≈ 4.4

The mean of her customer rating ≈ 4.4.

Objective #2: Understanding Frequency Distribution and the mean of grouped data.

When work with a large amount of data, when can group the data into classes of the same length. The number of tallys in each group will be the frequency. What we are creating is called a frequency distribution. We can then find the midpoint of each class and use that to calculate the grouped mean. The grouped mean will not quite be the same as the ungrouped mean, but it will be close. In some cases where we only have the frequency distribution and not the individual data values, then we have to use the grouped mean.

In the last section, we actually created a frequency distributiion when we built a bar, line, and circle graph:

Use the following data create a frequency distribution.

Ex. 10   A company surveyed its employees about how many miles to the gallon the car gets. Here are the results:

39, 20, 37, 28, 29, 17, 15, 42, 30, 26, 23, 32, 28, 20, 43, 38, 33, 20, 24, 19, 31, 27, 21, 27, 34, 31, 25, 22, 41, 35, 21, 36, and 29.

Solution:

Since the smallest number 15 and the largest number is 43, we can group data with class width of 5 units. So, our class intervals are:

15 – 19, 20 – 24, 25 – 29, 30 – 34, 35 – 39, 40 – 44.

It is important to make the class intervalses the same size so we can make accurate comparsion. Next, we need to count the number of

"hits" we have in each interval and tally the mark:

Class Intervals	Tally	Frequency, F
15 – 19	|||	3
20 – 24	‖‖‖ |||	8
25 – 29	‖‖‖ |||	8
30 – 34	‖‖‖		6
35 – 39	‖‖‖	5
40 – 44	|||	3

To calculate the mean of the grouped data, we will follow this procedure:

Find the Mean of grouped data in a frequency distribution:

1)   Find the midpoint M of each interval by averaging the endpoints.
2)   In each row, multiply the frequency with the midpoint. (FM products)
3)   Find the sum of the frequencies (n) and the sum of the FM products
4)   Divide the sum of the FM products by the sum of the frequencies to get the grouped mean.

Find the group mean of the following:

Ex. 11

Class Intervals	Tally	Frequency, F
15 – 19	|||	3
20 – 24	‖‖‖ |||	8
25 – 29	‖‖‖ |||	8
30 – 34	‖‖‖		6
35 – 39	‖‖‖	5
40 – 44	|||	3

Solution:

1) Compute the midpoint of each intervals:

$(15 + 19) \div 2 = 34 \div 2 = 17$ $(20 + 24) \div 2 = 44 \div 2 = 22$

$(25 + 29) \div 2 = 54 \div 2 = 27$ $(30 + 34) \div 2 = 64 \div 2 = 32$

$(35 + 39) \div 2 = 74 \div 2 = 37$ $(40 + 44) \div 2 = 84 \div 2 = 42$

Class Intervals	Tally	Frequency, F	Midpoint, M
15 – 19	|||	3	17
20 – 24	⊦⊦⊦⊦ |||	8	22
25 – 29	⊦⊦⊦⊦ |||	8	27
30 – 34	⊦⊦⊦⊦ |	6	32
35 – 39	⊦⊦⊦⊦	5	37
40 – 44	|||	3	42

2) Multiply F and M in each row.

Class Intervals	Tally	Frequency, F	Midpoint, M	FM Products
15 – 19	|||	3	17	3(17) = 51
20 – 24	⊦⊦⊦⊦ |||	8	22	8(22) = 176
25 – 29	⊦⊦⊦⊦ |||	8	27	8(27) = 216
30 – 34	⊦⊦⊦⊦ |	6	32	6(32) = 192
35 – 39	⊦⊦⊦⊦	5	37	5(37) = 185
40 – 44	|||	3	42	3(42) = 126

3) Find the sum of the frequencies and the FM products:

Class Intervals	Tally	Frequency, F	Midpoint, M	FM Products
15 – 19	|||	3	17	3(17) = 51
20 – 24	┼┼┼┼ |||	8	22	8(22) = 176
25 – 29	┼┼┼┼ |||	8	27	8(27) = 216
30 – 34	┼┼┼┼ |	6	32	6(32) = 192
35 – 39	┼┼┼┼	5	37	5(37) = 185
40 – 44	|||	3	42	3(42) = 126

n = 33   Sum FM = 946

4)   Divide Sum FM by n:

946 ÷ 33 = 28.66... ≈ 29 mpg

Hence, the grouped mean is approximately 29 mpg.
The actual ungrouped mean is 28.57... mpg which very close to the grouped mean.

Ex. 12   Bower's Books and Breakfast Company is conducting a survey of their online customers. One of the questions asks the customers to select which age range they fall under. The following frequency distribution shows the results for that particular question:

Class Intervals	Frequency, F	Class Intervals	Frequency, F
18 – 25	75	58 – 65	121
26 – 33	89	66 – 73	43
34 – 41	104	74 – 81	15
42 – 49	168	82 – 89	2
50 – 57	175	90 +	0

Solution:

1) Compute the midpoint of each intervals:

$(18 + 25) \div 2 = 43 \div 2 = 21.5$ $(26 + 33) \div 2 = 59 \div 2 = 29.5$

$(34 + 41) \div 2 = 75 \div 2 = 37.5$ $(42 + 49) \div 2 = 91 \div 2 = 45.5$

$(50 + 57) \div 2 = 107 \div 2 = 53.5$ $(58 + 65) \div 2 = 123 \div 2 = 61.5$

$(66 + 73) \div 2 = 139 \div 2 = 69.5$ $(74 + 81) \div 2 = 155 \div 2 = 77.5$

$(82 + 89) \div 2 = 171 \div 2 = 85.5$ 90 + is not applicable since the frequency is 0.

Class Intervals	Frequency, F	Midpoint, M
18 – 25	75	21.5
26 – 33	89	29.5
34 – 41	104	37.5
42 – 49	168	45.5
50 – 57	175	53.5
58 – 65	121	61.5
66 – 73	43	69.5
74 – 81	15	77.5
82 – 89	2	85.5

2)   Multiply F and M in each row.

Class Int.	Frequency, F	Midpoint, M	FM Products
18 – 25	75	21.5	1612.5
26 – 33	89	29.5	2625.5
34 – 41	104	37.5	3900
42 – 49	168	45.5	7644
50 – 57	175	53.5	9362.5
58 – 65	121	61.5	7441.5
66 – 73	43	69.5	2988.5
74 – 81	15	77.5	1162.5
82 – 89	2	85.5	171

3)   Find the sum of the frequencies and the FM products:

Class Int.	Frequency, F	Midpoint, M	FM Products
18 – 25	75	21.5	1612.5
26 – 33	89	29.5	2625.5
34 – 41	104	37.5	3900
42 – 49	168	45.5	7644
50 – 57	175	53.5	9362.5
58 – 65	121	61.5	7441.5
66 – 73	43	69.5	2988.5
74 – 81	15	77.5	1162.5
82 – 89	2	85.5	171

n = 792   Sum FM = 36908

4)   Divide Sum FM by n:

36908 ÷ 792 = 46.6... ≈ 47 years old

Hence, the grouped mean is approximately 47 years old.