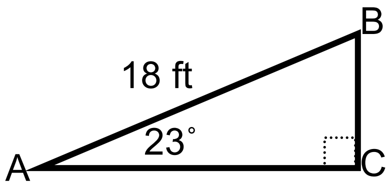
Sect 10.3 – Solving Right Triangles

Objective 1: Finding the missing sides and angles of a right triangle.

Given either two sides or a side and one of the acute angles of a right triangle, we can use our trigonometric ratios and the Pythagorean Theorem to solve for the missing sides and angles of a right triangle.

Finding the missing sides and angles of the following. Round all sides to the nearest tenth and all angles to the nearest minute:

Ex. 1



Solution:
Since ∠A and ∠B are complimentary, then
m∠B = 90° – 23° = 67°.
We have the hypotenuse and the acute angle A. We can use the sine of 23° to find BC and the cosine of 23° to find AC.

$\sin 23^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{18}$

To solve, multiply by 18:

BC = 18•sin 23°

= 18•0.3907… = 7.03…
≈ 7.0 ft

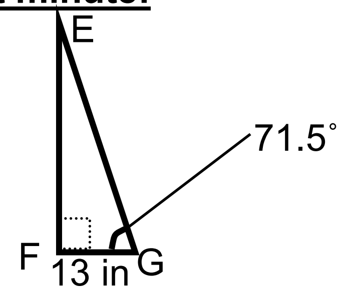
$\cos 23^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{AC}{18}$

To solve for BC, multiply by 18:

AC = 18•cos 23°

= 18•0.9205… = 16.56…
≈ 16.6 ft
Thus, BC ≈ 7.0 ft. AC ≈ 16.6 ft and m∠B = 67°.

Ex. 2



Solution:
Since ∠E and ∠G are complimentary, then
m∠E = 90° – 71.5° = 18.5° = 18°30'.
We have the adjacent side and and the acute angle G. We can use the tangent of 71.5° to find the EF and the cosine of 71.5° to find EG.

$\tan 71.5^\circ = \frac{\text{opp}}{\text{adj}} = \frac{EF}{13}$

To solve, multiply by 13:

EF = 13•tan 71.5°

= 13•2.98… = 38.85…
≈ 38.9 in

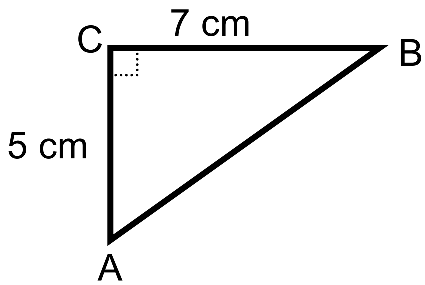
$\cos 71.5^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{13}{EG}$

To solve for EG, multiply by EG and then divide by cos 71.5°:

EG•cos 71.5° = 13

EG = 13/cos 71.5° =13/0.317…
= 40.97… ≈ 41.0 in
Hence, EF ≈ 38.9 in, EG ≈ 41.0 in, and m∠E = 18°30'.

Ex. 3



Solution:

We have the two legs of a right triangle. We can use the inverse tangent function to find ∠B: .

$\tan B = \frac{\text{opp}}{\text{adj}} = \frac{5}{7}$

Thus, $B = \tan^{-1}(\frac{5}{7})$

$= 35.53\ldots^\circ$

Convert into DMS and round:

35˚32'15"6 ≈ 35˚32'

Thus, m∠B = 35˚32'

Since ∠A and ∠B are complimentary, then

m∠A = 90˚ – 35˚32' = 54˚28'

To find the length AB, we can use the Pythagorean Theorem:

$(AB)^2 = (7)^2 + (5)^2$

$(AB)^2 = 49 + 25$

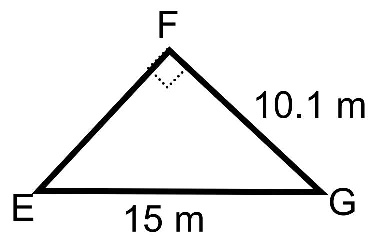
$(AB)^2 = 74$

$AB = \sqrt{74} = 8.60\ldots \approx 8.6$ cm

Thus, m∠B ≈ 35˚32',

m∠A ≈ 54˚28', and AB ≈ 8.6 cm.

Ex. 4



Solution:

We have the leg and the hypotenuse of a right triangle. we can use the inverse cosine to to find ∠G.

$\cos G = \frac{\text{adj}}{\text{hyp}} = \frac{10.1}{15}$

Hence, $G = \cos^{-1}(\frac{10.1}{15})$

$= 47.67\ldots^\circ$

Convert into DMS and round:

47˚40'30"5 ≈ 47˚31'

Therefore, m∠G = 47˚31'

Since ∠E and ∠G are complimentary, then

m∠E = 90˚ – 47˚31' = 42˚29'

To find the length EF, we can use the Pythagorean Theorem:

$(10.1)^2 + (EF)^2 = (15)^2$

$102.01 + (EF)^2 = 225$

$\underline{-\,102.01 \qquad = -\,102.01}$

$(EF)^2 = 122.99$

$EF = \sqrt{122.99} = 11.09\ldots \approx 11.1$ m

Hence, m∠G ≈ 47˚31',

m∠E = 42˚29', and EF ≈ 11.1 m.

Objective 2: Solving applications involving right triangles.

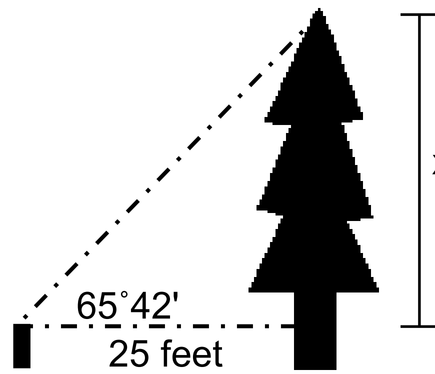
The key to solving applications involving right triangles is drawing a reasonable diagram to represent the situation. Next, we need determine what information we are given and what information we need to find. Finally, we then use the appropriate trigonometric function to solve the problem.

Solve the following:

Ex. 5 A surveyor measures the angle between her instruments and the top of a tree and finds the angle of elevation to be 65˚42'. If her instruments are 4 ft high off of the ground and are 25 feet away from the center of the base of the tree and the ground is leveled, how high is the tree? Round to three significant digits.

Solution:

We begin by drawing the surveyor's instruments and the tree. Since her instruments are 4 ft above the ground, we can draw a horizontal line from her her instruments to four feet above the center of the base of the tree. Drawing a line from the top of the tree to her instruments and a vertical line down from the top of the tree four feet above the center of the base of the tree, we get a right triangle. The adjacent side is 25 feet and we are looking for the opposite of the triangle. Once we get the opposite side, we will need to add 4 ft to the answer to find the height of the tree. The trigonometric ratio that uses the opposite and adjacent sides is the tangent function. Hence,



$\tan 65^\circ 42' = \frac{\text{opp}}{\text{adj}} = \frac{x}{25}$ (multiply by 25 to solve for x)

$x = 25\bullet\tan 65^\circ 42' = 25\bullet 2.214\ldots = 55.36\ldots$ ft

Now, add 4 ft to find the height of the tree:

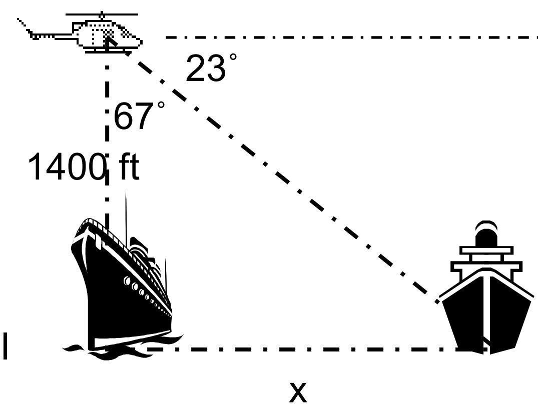
$55.36\ldots + 4 = 59.36 \approx 59.4$ ft

So, the tree is approximately 59.4 ft tall.

Ex. 6 A helicopter flying directly above an ocean liner at a height of 1400 ft, locates another boat at a 23˚ angle of depression. How far apart are the two ships? Round to the nearest hundred feet.

Solution:

The angle of depression is the angle between a horizontal line and the line of observation. In other words, it is the angle one has to look down to see the observed object, which is a ship in this case. The helicopter is flying directly above the ocean liner. We can draw a vertical line between the helicopter and the ocean liner. Next we can draw a line from the helicopter to the other ship. The angle between



this two lines and the angle of depression are complimentary angles. Thus, the angle between the two lines is 90˚ – 23˚ = 67˚. Now, drawing a line between the two ships creates a right triangle. The vertical height is the adjacent side of 67˚ and we are looking for the opposite side of 67˚. So, we will need to use the tangent function.

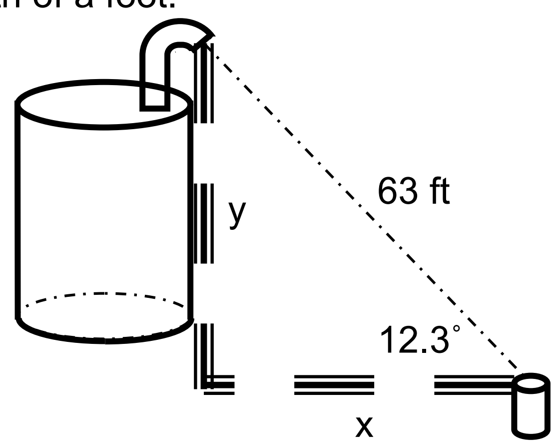
$\tan 67^\circ = \frac{\text{opp}}{\text{adj}} = \frac{x}{1400}$ (multiply by 1400 to solve for x)

$x = 1400 \bullet \tan 67^\circ = 1400 \bullet 2.35\ldots = 3298.1\ldots \text{ ft} \approx 3300 \text{ ft}$

So, the ships are approximately 3300 ft apart.

Ex. 7 The distance from a water source to the input of a tank 63 ft. If the angle of elevation is 12.3˚ from the source to the input of the tank, how much pipeline will be needed if the plumber is planning to run the pipe horizontally two feet under the ground from the water source to right below the input of tank and then turn the pipe vertically and run it to the input? Round to the nearest tenth of a foot.

Solution:



We begin by drawing the tank and the water source. We draw a line from the input of the tank to the water source. We then draw two lines for where the pipe will run. The two lines for the pipe are the legs of a right triangle and the 63 ft is the length of the hypotenuse. The angle of elevation is the acute angle we will be using. To find the opposite side (y), we will need to use the sine function and to find the adjacent side, we will need to use the cosine function.

$\sin 12.3^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{y}{63}$ (multiply by 63 to solve for y)

$y = 63 \bullet \sin 12.3^\circ = 63 \bullet 0.213\ldots = 13.42\ldots \text{ ft}$

$\cos 12.3^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{x}{63}$ (multiply by 63 to solve for x)

$x = 63 \bullet \cos 12.3^\circ = 63 \bullet 0.977\ldots = 61.55\ldots \text{ ft}$

Now, add the two results:
x + y = 61.55… + 13.42… = 74.97… ≈ 75.0 ft
The plumber will need 75.0 ft of pipe.

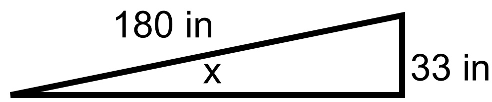
Ex. 8 A new ordinance states that the incline of a handicapped access can be no more than 9˚. Currently, a 15-ft long ramp leading into a building climbs 2 ft 9 in. Does this ramp comply with the ordinance? If not, how long does the ramp need to be? Round to the nearest inch.

<u>Solution:</u>

First, let's convert 2 ft 9 in and 15 ft into inches:

$$2\text{ ft} + 9\text{ in} = \frac{2\text{ ft}}{1}\left(\frac{12\text{ in}}{1\text{ft}}\right) + 9\text{ in} = 24\text{ in} + 9\text{ in} = 33\text{ in}$$

$$15\text{ ft} = \frac{15\text{ft}}{1}\left(\frac{12\text{ in}}{1\text{ft}}\right) = 180\text{ in}$$

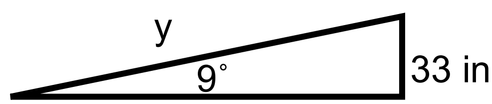


Since the ramp climbs 33 in, then the height of the ramp is 33 in. This will correspond to the opposite side of the incline. The length of the ramp is 180 in. This will correspond to the hypotenuse. This means we will need to use the inverse sine function to find the angle.

$$\sin x^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{33}{180} \qquad \text{(use the inverse sine)}$$

$$x = \sin^{-1}\left(\frac{33}{180}\right) = 10.56\ldots^\circ \text{ which is greater than } 9^\circ$$

Thus, the current ramp does not comply with the ordinance.



Since the ramp climbs 33 in, then the height of the ramp is 33 in. This will correspond to the opposite side of the incline. The length of the ramp is length of the ramp which is the hypotenuse. This means we will need to use the sine function.

$$\sin 9^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{33}{y} \qquad \text{(multiply by y)}$$

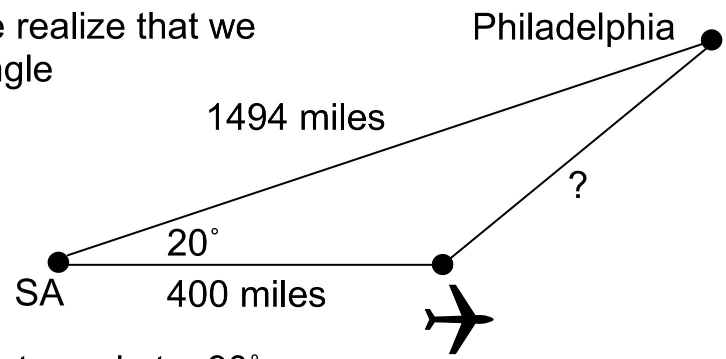
$$y \sin 9^\circ = 33 \qquad \text{(divide by } \sin 9^\circ\text{)}$$

$$y = \frac{33}{\sin 9^\circ} = \frac{33}{0.156...} = 210.95\ldots \approx 211\text{ in}$$

But, 211 ÷ 12 = 17 with a remainder 7, so
211 in = 17 ft 7 in
The ramp should be 17 ft 7 in to comply with the ordinance.

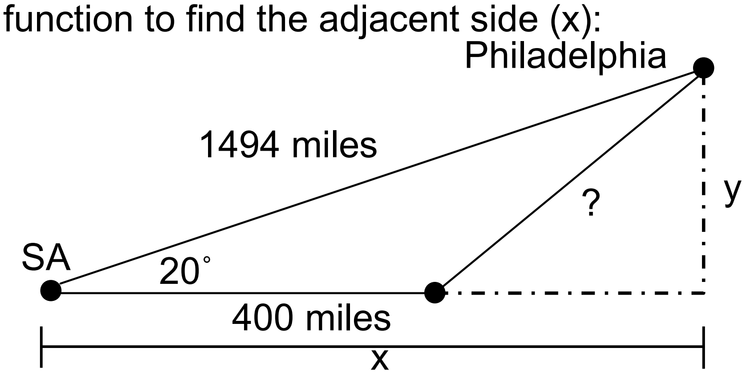
Ex. 9 Two cities, San Antonio and Philadelphia, are 1494 miles apart. To avoid a bad storm, a pilot took a course that was 20˚ further south from San Antonio for 400 miles and then turned to fly towards Philadelphia. How much further did the pilot have to fly to reach Philadelphia? Round to the nearest mile.

<u>Solution:</u>



If we draw the picture, we realize that we may not have a right triangle since when the pilot turned the plane, there is no guarantee that he or she turned at a 90˚ angle. Let's suppose that the pilot had went further and then turned at a 90˚.

We can use the cosine function to find the adjacent side (x):



$\cos 20^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1494}$

(multiply by 1494)

x = 1494•cos 20˚

x = 1494•0.939…

x = 1403.9… mi

The extra distance the pilot would have had to travel in order to turn at a 90˚ is 1403.9… – 400 = 1003.9… miles.

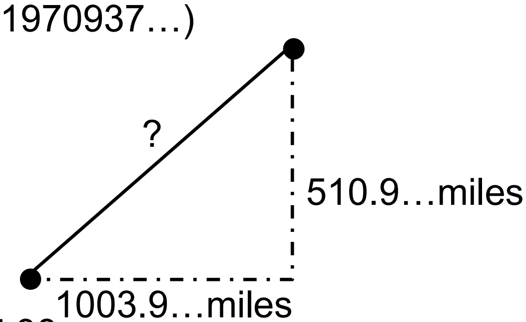
Now use the Pythagorean Theorem to find the opposite side (y):

$y^2 + (1403.9\ldots)^2 = (1494)^2$

$y^2 + 1970937\ldots = 2232036$ (subtract 1970937…)

$y^2 = 261098\ldots$

$y = \sqrt{261098...} = 510.97\ldots$ mi



Now consider the smaller right triangle in the diagram. We know both legs so we can use the Pythagorean Theorem to find ?

$?^2 = (1003.9\ldots)^2 + (510.9\ldots)^2$

$?^2 = 1007816\ldots + 261098\ldots = 1268915.38$

$? = \sqrt{1268915.38} = 1126.4\ldots$

So, the plane traveled 400 + 1126.4… = 1526.4… miles.

Thus, 1526.4 – 1494 = 32. 46… ≈ 32 miles

So, the plane traveled 32 miles further.