Sect 10.1 – Angles and Triangles

Objective 1: Converting Angle Units

Each degree can be divided into 60 minutes, denoted 60', and each minute can be divided into 60 seconds, denoted 60". Hence, $1^\circ = 60'$ and $1' = 60''$. In most technical applications, the measure of angles will be rounded to the nearest minute and in most trade applications, the measure of angles will be rounded to the nearest degree.

To convert an angle written as a decimal or mixed number into an angle written in degrees and minutes, multiply the decimal part or proper fraction part by $\frac{60'}{1^\circ}$. The result will be the number of minutes.

Convert the following angles into degrees and minutes:

Ex. 1a $35\frac{2}{5}^\circ$ Ex. 1b 98.3°

Ex. 1c 67.35° Ex. 1d $17\frac{3}{8}^\circ$

Solution:

a) $35\frac{2}{5}^\circ = 35^\circ + \frac{2}{5}^\circ \bullet \frac{60'}{1^\circ} = 35^\circ 24'$

b) $98.3^\circ = 98^\circ + 0.3^\circ \bullet \frac{60'}{1^\circ} = 98^\circ 18'$

c) $67.35^\circ = 67^\circ + 0.35^\circ \bullet \frac{60'}{1^\circ} = 67^\circ 21'$

d) $17\frac{3}{8}^\circ = 17^\circ + \frac{3}{8}^\circ \bullet \frac{60'}{1^\circ} = 17^\circ 22.5'$

To convert an angle written in degrees and minutes into an angle written as a decimal or mixed number, multiply the minutes by $\frac{1^\circ}{60'}$ and add the result to the degrees.

Convert the following angles into decimal form:

Ex. 2a $38^\circ 36'$ Ex. 2b $107^\circ 12'$

Ex. 2c $7^\circ 9'$ Ex. 2d $27'$

Solution:

a) $38^\circ 36' = 38^\circ + 36'\left(\frac{1^\circ}{60'}\right) = 38^\circ + 0.6^\circ = 38.6^\circ$

b) $107^\circ 12' = 107^\circ + 12'\left(\frac{1^\circ}{60'}\right) = 107^\circ + 0.2^\circ = 107.2^\circ$

c) $7^\circ 9' = 7^\circ + 9'\left(\frac{1^\circ}{60'}\right) = 7^\circ + 0.15^\circ = 7.15^\circ$

d) $27' = 27'\left(\frac{1^\circ}{60'}\right) = 0.45^\circ$

To convert angles on a scientific calculator, we will be using the DD→DMS and DMS→DD keys on a calculator. On a TI-30XA, the DD→DMS is the yellow above the + key and the DMS→DD is the yellow above the = key.

To convert an angle written as a decimal or a mixed number into an angle written in degrees and minutes,

1) Type the angle. (ex. 43.755°)

2) Hit $\boxed{\text{2nd}}$ $\boxed{=}$. The answer should read 43˚45' 18" 0.

Caution. Do not hit the equal signs again. If you hit the equal signs again, the calculator will convert the angle back into decimal form. The last digit on the calculator display is the fraction of seconds in an angle

Let's redo example #1 on our scientific calculator.

Angle in Degrees	Keystrokes (DD→DMS)	Display Answer
$35\frac{2}{5}^\circ$	$\boxed{\text{2nd}}$ $\boxed{=}$	35˚24' 00"0
98.3°	$\boxed{\text{2nd}}$ $\boxed{=}$	98˚18' 00"0
67.35°	$\boxed{\text{2nd}}$ $\boxed{=}$	67˚21' 00"0
$17\frac{3}{8}^\circ$	$\boxed{\text{2nd}}$ $\boxed{=}$	17˚22' 30"0

Notice with the last angle, the calculator automatically converted the 0.5 minutes into 30 seconds.

To convert an angle written in degrees and minutes into an angle written as a decimal or mixed number,

1) Type the angle in the following form D.MMSSs (ex. 58˚24'9" would be written as 58.2409)

2) Hit $\boxed{\text{2nd}}$ $\boxed{+}$. The answer should read 58.4025.

Caution. The minutes and seconds must be written as a two-digit number. Any fraction of a second is written as a one digit after the seconds. So, if the angle is 72˚3'4.5", we would type 72.03045 and hit $\boxed{\text{2nd}}$ $\boxed{+}$. The display answer will be 72.05125˚.

Let's redo example #2 on our scientific calculator.

Angle in Degrees & Minutes	Keystrokes (DMS→DD)	Display Answer
38˚36'	38.36 $\boxed{\text{2nd}}$ $\boxed{+}$	38.6˚
107˚12'	107.12 $\boxed{\text{2nd}}$ $\boxed{+}$	107.2˚
7˚9'	7.09 $\boxed{\text{2nd}}$ $\boxed{+}$	7.15˚
27'	0.27 $\boxed{\text{2nd}}$ $\boxed{+}$	0.45˚

Note, if you enter an angle measurement with 60 or more minutes, the calculator will automatically convert the angle to the next higher degree. Thus, if the angle is 43˚75', we enter 43.75 and hit the DMS→DD key, we get 44.25˚. If we then hit the DD→DMS key, we get 44˚15'. Also, to add or subtract two angles in Degrees-Minutes-Seconds, we enter each angle and convert them into decimals and perform the operation. Then, we convert the result back into degrees-minutes-seconds.

Perform the following operations on a scientific calculator:

Ex. 3a 43˚48'25" + 56˚25'54" Ex. 3b 98˚14'7" – 14˚8'29"

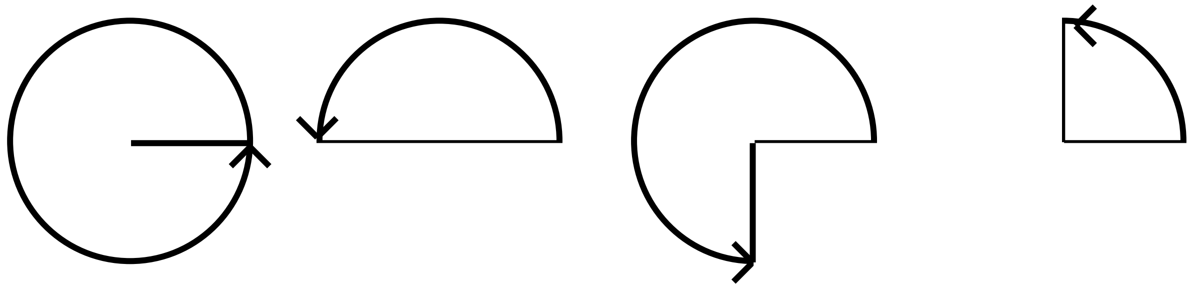
Solution:

a) Enter 43.4825 and hit the DMS→DD key + 56.2554 and hit the DMS→DD key =. The display should read 100.2386… Hit DD→DMS key. The final answer is 100˚14'19".

b) Enter 98.1407 and hit the DMS→DD key – 14.0829 and hit the DMS→DD key =. The display should read 84.0938... Hit DD→DMS key. The final answer is 84˚5'38"

Objective 2: Understanding Angle Measurement in Radians.

So far in geometry, we have measured all of the angles in units called degrees. We now want to introduce a new unit for measuring angles called radians. Consider a circle with a radius of 1 unit (called the unit circle).



For an angle at the center of the circle to sweep out a full circle, it must make one full rotation or 360˚. The circumference of a unit circle is C = 2π(1) = 2π. So, we will set 360˚ equal to 2π radians (abbreviated rad). If our angle sweeps out half of a unit circle, it makes half of a rotation or 180˚. The circumference of half of a unit circle is C = $\frac{1}{2}$(2π(1)) = π. Thus, we will set 180˚ equal to π rad. Similarly, if we go three-quarters of a rotation, the angle will be 270˚ = $\frac{3\pi}{2}$ rad and if we go one-quarter of a rotation, the angle will be 90˚ = $\frac{\pi}{2}$ rad. Thus, the corresponding angle in radians will equal the length of the portion of the circumference of the unit circle that is swept out by the angle.

In general, we will use the fact that 180˚ = π rad to convert between degrees and radians.

<u>**Convert the following angles to radians. Round to four significant digits:**</u>

Ex. 4a 75.36˚ Ex. 4b $43\frac{2}{3}^{\circ}$

Ex. 4c 17˚15' Ex. 4d 85˚9'

<u>Solution:</u>

a) $75.36^\circ = 75.36^\circ \bullet \frac{\pi}{180^\circ} = 1.3152\ldots \approx 1.315$ rad

b) $43\frac{2}{3}^\circ = 43\frac{2}{3}^\circ \bullet \frac{\pi}{180^\circ} = 0.76212\ldots \approx 0.7621$ rad

c) $17^\circ 15' = 17^\circ + 15'\left(\frac{1^\circ}{60'}\right) = 17^\circ + 0.25^\circ = 17.25^\circ \bullet \frac{\pi}{180^\circ} = 0.30106\ldots$
≈ 0.3011 rad

d) $85^\circ 9' = 85^\circ + 9'\left(\frac{1^\circ}{60'}\right) = 85^\circ + 0.15^\circ = 85.15^\circ \bullet \frac{\pi}{180^\circ} = 1.4861\ldots$
≈ 1.486 rad

Convert the following angles to degrees. Round to four significant digits:

Ex. 5a 0.2562 rad

Ex. 5b $\frac{5\pi}{3}$ rad

Ex. 5c $\frac{\pi}{12}$ rad

Ex. 5d 1 rad

Solution:

a) $0.2562 \text{ rad} = 0.2562 \bullet \frac{180^\circ}{\pi} = 14.679\ldots \approx 14.68^\circ$

b) $\frac{5\pi}{3} \text{ rad} = \frac{5\pi}{3} \bullet \frac{180^\circ}{\pi} = 300.0^\circ$

c) $\frac{\pi}{12} \text{ rad} = \frac{\pi}{12} \bullet \frac{180^\circ}{\pi} = 15.00^\circ$

d) $1 \text{ rad} = 1 \bullet \frac{180^\circ}{\pi} = 57.295\ldots \approx 57.30^\circ$

To convert angles on a scientific calculator, you first need to know what angle mode your calculator is in. Look on the calculator screen, you will see "Deg" or "D" if you are in degree mode, "Rad" or "R" if you are in radian mode, and "Gra" or "G" if you are in gradient mode. To change modes on TI-30XA, hit the DRG key right next to the 2nd key (you may need to hit it twice to get to the correct mode). You will notice the display will cycle through Deg→Rad→Gra if you keep hitting the DRG key. To convert between the degrees and radians, you need to use the DRG ▸ (hit 2nd and then the DRG on a TI-30XA). To convert from degrees to radian, be sure you are in deg mode, enter the angle on the calculator and hit 2nd and then the DRG. Let's try some examples:

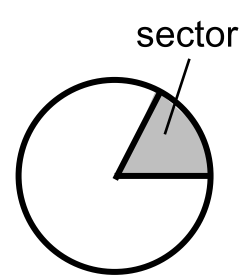
Angles in Degrees (Be sure you're in Deg mode before you start)	**Keystrokes**	**Display Answer in Radians**
35.27°	**2nd** **DRG**	**0.615577627 rad**
189.79°	**2nd** **DRG**	**3.312460387 rad**
57.295°	**2nd** **DRG**	**0.999986395 rad**
121.56°	**2nd** **DRG**	**2.121622239 rad**

To convert from radians to degrees, be sure you are in rad mode, enter the angle on the calculator and hit 2nd and then the DRG twice. Let's try some examples:

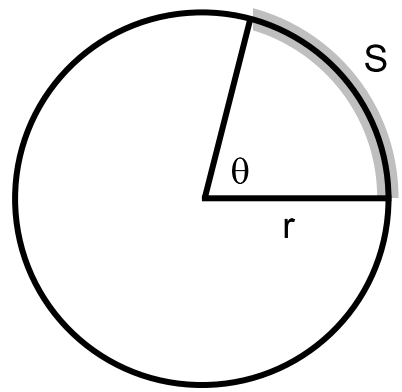
Angles in Radians (Be sure you're in Rad mode before you start)	Keystrokes	Display Answer in Degrees
0.8695 rad	2nd DRG 2nd DRG	49.81868029°
$\frac{5\pi}{6}$ rad	2nd DRG 2nd DRG	150°
4.241 rad	2nd DRG 2nd DRG	242.9914009°
1.25π rad	2nd DRG 2nd DRG	225°

Objective 3: Calculating the arc length and the area of a sector.

A **Sector** of a circle is the portion of the circle enclosed by two radii of the circle and the intersected edge of the circle. The intersected edge of the circle is called the **arc** and the angle between the two radii of the circle is called the central angle. We will denoted the central angle by the Greek letter theta: θ.



Properties of Sectors:



Let r = the radius of the circle
θ = the measure of the central angle measured in radians
Then
The arc length: $S = r\theta$
The area of the sector: $A = \frac{1}{2}r^2\theta$

Note: If we have a full circle, then $\theta = 360^\circ = 2\pi$ rad. Thus, $S = r\theta = r(2\pi) = 2\pi r$ which is the formula for the circumference of the circle, and $A = \frac{1}{2}r^2\theta = \frac{1}{2}r^2(2\pi) = \pi r^2$ which is the formula for the area of the circle.

Find the arc length and the area of the following:

Ex. 6a A sector with central angle 0.750π rad and radius 18.0 ft.

Ex. 6b A sector with central angle 78° and radius 4.52 in.

Solution:

a) The angle is already in radians. So, we just plug into the formulas:
$S = r\theta = (18)(0.75\pi) = 42.41\ldots \approx 42.4$ ft
$A = \frac{1}{2}r^2\theta = \frac{1}{2}(18)^2(0.75\pi) = \frac{1}{2}(324)(0.75\pi) = 381.7\ldots \approx 382\ \text{ft}^2$

b) We first need to convert the angle to radians:
$78^\circ = 1.3613\ldots$ rad
Now, plug into the formulas:
$S = r\theta = (4.52)(1.3613\ldots) = 6.15\ldots \approx 6.2$ in
$A = \frac{1}{2}r^2\theta = \frac{1}{2}(4.52)^2(1.3613\ldots) = \frac{1}{2}(20.4304)(1.3613\ldots)$
$= 13.9\ldots \approx 14\ \text{in}^2$

Objective 4: Calculating Linear and Angular Speed.

The relationship between distance, rate, and time is distance = rate $\times$ time. If we late d be the distance, v be the rate, and t be the time, we can write this relationship as $d = vt$. To solve for v, we would divide both sides by t to get $v = \frac{d}{t}$. In this relationship, the rate is referred to as the **average linear**

speed. We can apply the same principle to objects that are moving along an arc of a circle. Let θ be the angle the object sweeps out while traveling along an arc measured in radians and t be the time, then the average angular speed $w = \frac{\theta}{t}$

Average Linear Speed

$v = \frac{d}{t}$

Average Angular Speed

$w = \frac{\theta}{t}$, where θ is in radians

Solve the following:

Ex. 7 The blades of a fan have a radius of 8.25 in and rotates 37.0 revolutions every three seconds.

a) Find the linear speed of the tip of the blade.

b) Find the angular speed.

Solution:

a) We first need to find the distance the tip of the blade travelled: Since it performed 37 revolutions, then the distance it traveled is 37 times the circumference of the circle.

$$C = 2\pi r = 2\pi(8.25) = 16.5\pi = 51.8362\ldots \text{ in}$$

Thus, 37 times C is:

$$37 \bullet C = 37 \bullet 51.8362\ldots = 1917.94\ldots \text{ in}$$

So, the total distance traveled is 1917.94 inches in three seconds. Now, we can calculate the average linear speed:

$$v = \frac{d}{t} = \frac{1917.9\ldots}{3} = 639.3\ldots \approx 639 \text{ in/sec}$$

The tip of the blades linear speed is 639 in/sec.

b) In rotating 37 times, the angle swept out is $37 \bullet 2\pi = 74\pi = 232.47\ldots$ rad in 3 seconds.

So, the average angular speed is:

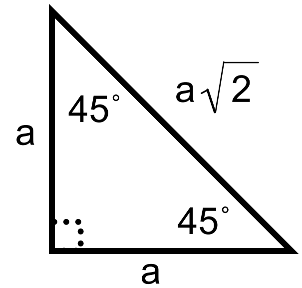
$$w = \frac{\theta}{t} = \frac{232.47\ldots}{3} = 77.49\ldots \approx 77.5 \text{ rad/sec}$$

The angular speed is 77.5 rad/sec.

Objective 5: Understanding Special Right Triangles.

The first special right triangle we want to examine is a 45˚-45˚-90˚ right triangle. Since two of the angles are the same, that means two of the sides are equal, so we have an isosceles triangle. Also, the two equal sides of the isosceles triangle are the legs of the right triangle since the hypotenuse is always opposite of the right angle. If we let a be the length of one of the equal sides, then we can use the Pythagorean Theorem to find the hypotenuse.

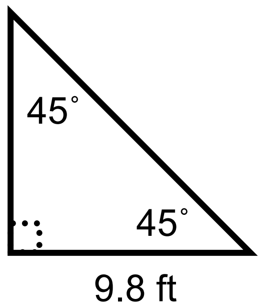
$c^2 = a^2 + a^2$ (combine like terms)
$c^2 = 2a^2$ (take the square root)
$c = \sqrt{2a^2}$
$c = a\sqrt{2}$



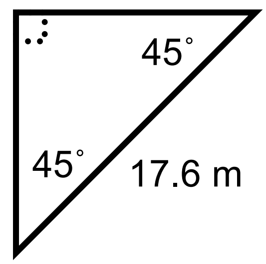
This means that the ratio of sides for a 45˚-45˚-90˚ is a: a: $a\sqrt{2}$.

<u>Find the lengths of the sides of the following triangles:</u>

Ex. 8a



Ex. 8b



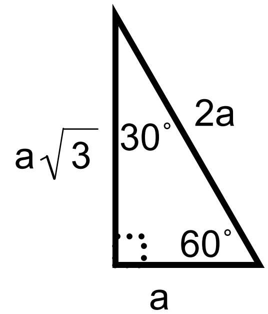
<u>Solution:</u>
Since one of the legs is 9.8 ft, then the other leg is also 9.8 ft.
The hypotenuse is $c = a\sqrt{2}$
$= 9.8\sqrt{2} = 13.8592\ldots \approx 14$ ft.
So, the sides are 9.8 ft, 9.8 ft, and 14 ft.

<u>Solution:</u>
The hypotenuse is 17.6 m, so,
$a\sqrt{2} = 17.6$ m. Now, solve for a:
$a\sqrt{2} = 17.6$ (divide by $\sqrt{2}$)
$a = 12.445\ldots \approx 12.4$ m
So, the sides are 12.4 m, 12.4 m, and 17.6 m.

The second special right triangle we want to examine is a 30˚-60˚-90˚ right triangle. If we take an equilateral triangle and cut it in half along the altitude, we get two 30˚-60˚-90˚ triangles. The shortest side is half of the longest side. If we let a be the length of the shortest side, then 2a will be the length of the longest or the hypotenuse. We can use the Pythagorean Theorem to find the other leg of the triangle.

$(2a)^2 = a^2 + h^2$ (simplify)
$4a^2 = a^2 + h^2$ (subtract a^2)
$\underline{-a^2 = -a^2}$
$3a^2 = h^2$

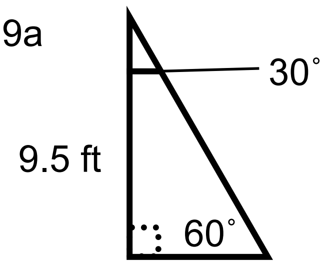
$h^2 = 3a^2$ (take the square root)
$h = \sqrt{3a^2} = a\sqrt{3}$



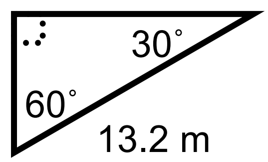
This means that the ratio of sides for a 30˚-60˚-90˚ is a: $a\sqrt{3}$: 2a.

Find the lengths of the sides of the following triangles:

Ex. 9a



Ex. 9b

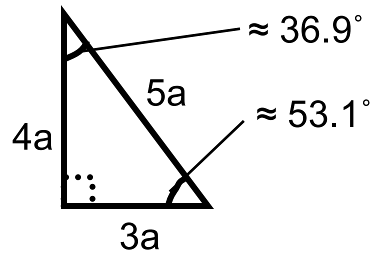


Solution:
Since the longer leg is 9.5 ft, then $a\sqrt{3}$ = 9.5 ft. Solving for a, we get:
$a\sqrt{3} = 9.5$ (divide by $\sqrt{3}$)
a = 5.484… ≈ 5.5 ft
The hypotenuse is 2a
= 2(5.484…) = 10.9… ≈ 11 ft
Thus, the sides are 5.5 ft, 9.5 ft, and 11 ft.

Solution:
Since the hypotenuse is 13.2 m, then 2a = 13.2 m. Solving for a, we get:
2a = 13.2 (divide by 2)
a = 6.60 m
The longer leg is $a\sqrt{3} = 6.6\sqrt{3}$
= 11.43… ≈ 11.4 m
Thus, the sides are 6.60 m, 11.4 m, and 13.2 m.

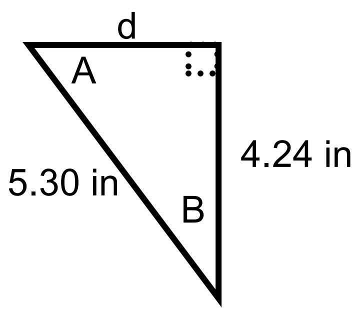
The third special right triangle is 3-4-5 triangle. If the ratios of the sides of the triangle are 3a: 4a: 5a, then the triangle is a right triangle since:
$(5a)^2 = (3a)^2 + (4a)^2$ (simplify)
$25a^2 = 9a^2 + 16a^2$ (combine like terms)
$25a^2 = 25a^2$

The angle opposite of shortest leg, 3a, is approximately 36.9˚ and the angle opposite the longer leg, 4a, is approximately 53.1˚.



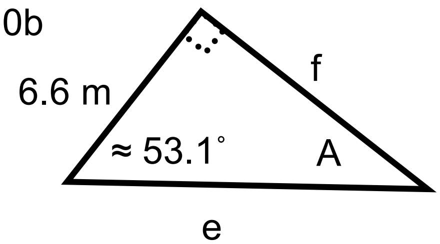
Find the missing sides and angles of the following:

Ex. 10a



Solution:
Since the ratio of 4.24: 5.3 is 4: 5, then we have a 3-4-5 triangle. Since the hypotenuse is 5.3 in, then
5a = 5.3 (divide by 5)
a = 1.06 in
The shortest leg is d = 3a = 3(1.06) = 3.18 in.
Since B is the angle opposite of the shortest side, then B ≈ 36.9˚ and A ≈ 53.1˚.
So, d = 3.18 in, A ≈ 53.1˚ and B ≈ 36.9˚.

Ex. 10b



Solution:
Since the other angles are 90˚ and 53.1˚, then A ≈ 36.9˚. Since 6.6 m is opposite of 36.9˚, it is the shortest side. So,
3a = 6.6 (divide by 3)
a = 2.2 m
The longer leg is f = 4a = 4(2.2) = 8.8 m.
The hypotenuse is e = 5a = 5(2.2) = 11 m.
So, f = 8.8 m, e = 11 m, and A ≈ 36.9˚.