ENGINEERING CALCULUS II
MTH 1212

Vectors & planes in space

by

NUR HILMI MOHAMAD (0134217)
http://www.geocities.com/hirumi_san

Vectors & planes in space

- How to express vector, v
  - using bracket, v = <x,y,z>
  - using i-j-k notation v = xi+yj+zk
- How to find vector & its magnitude
  - Find the difference between 2 points
    - A(a,b,c) to B(x,y,z)
    - AB = (x-a)i + (y-b)j + (z-c)k = <x-a,y-b,z-c>
    - |AB| = / (x-a)2 + (y-b)2 + (z-c)2
- Vector operation
  - Addition & subtraction
  - Multiplication
    - By scalar
    - Dot & cross
- Dot & cross product
  - a.b = |a||b| cos θ = ax*bx + ay*by + az*bz
  - a x b = [|a||b| sin θ]n = (ay*bz – az*by)i + ………
- Component & projection
  - $comp_b a$ = |a| cos θ
  - $proj_b a = comp_b a \ u_b$
- Plane equation
  - $P_0 P$.n = 0
  - P0(a,b,c) P(x,y,z) n=<A,B,C>
  - P0P = <x-a,y-b,z-c>
- Symmetric equation & parametric
  - x(t), y(t), z(t)   t = x-a / A = x-b / B = x-c / C

How to express vector

- using sharp bracket

  $\mathbf{v} = \langle a, b, c \rangle$

- using **i** - **j** - **k** notation

  $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$

How to evaluate vector

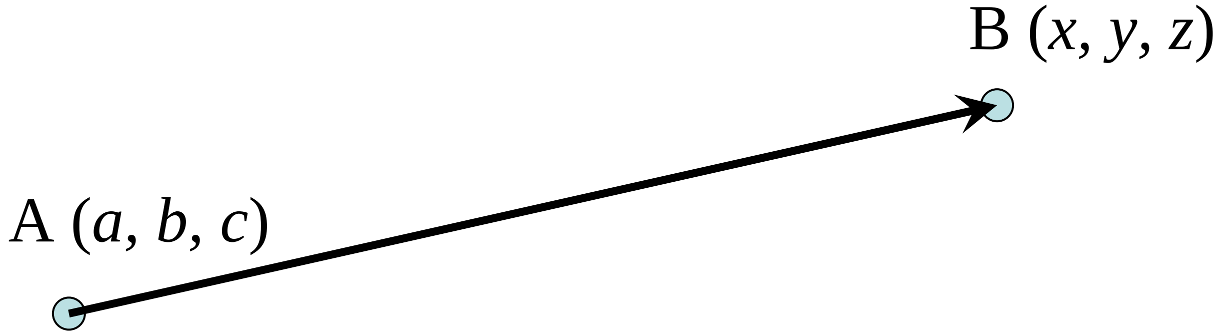
- When 2 points are given
- Find vector *AB*

$$\mathbf{v} = \overrightarrow{AB} = (x-a)\mathbf{i} + (y-b)\mathbf{j} + (z-c)\mathbf{k}$$

$$= \left\langle (x-a), (y-b), (z-c) \right\rangle$$

- Find its magnitude

$$\|\mathbf{v}\| = \left\|\overrightarrow{AB}\right\| = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$$



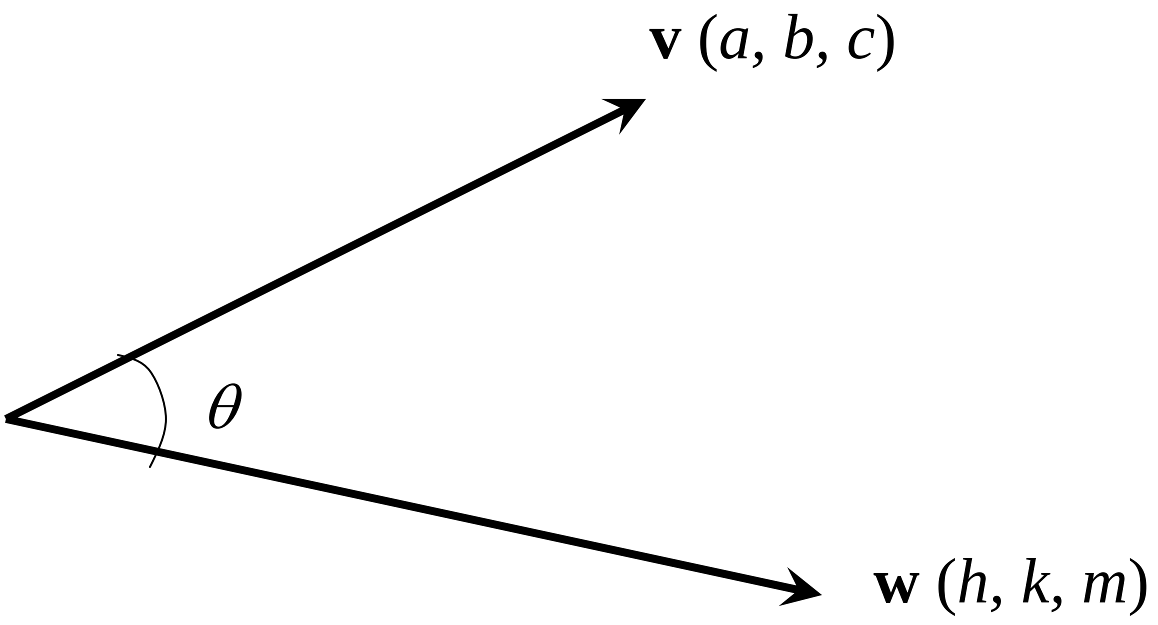
Unit vector

- Vector divided with its magnitude
- Showing the direction without scalar multiplication
- ||**u**|| is always 1

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

$$\|\mathbf{u}\| = 1$$

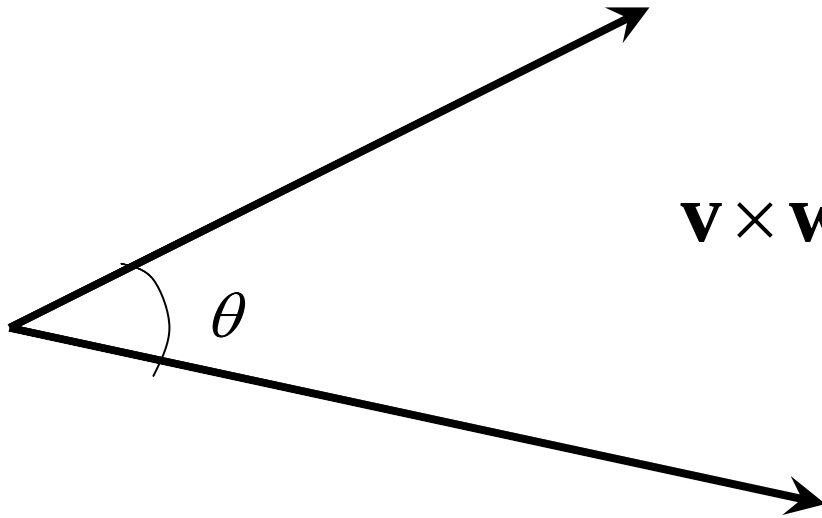
Dot product



$$\mathbf{v} \cdot \mathbf{w} = ah + bk + cm$$

$$\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \|\mathbf{w}\| \cos\theta$$

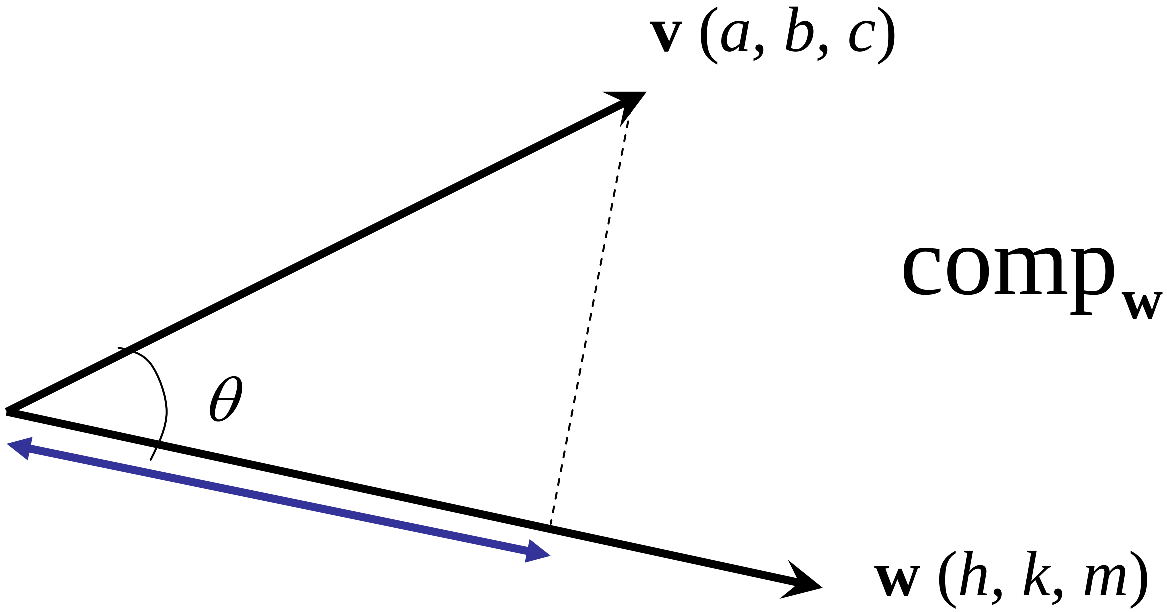
Cross product



$$\mathbf{v}\times\mathbf{w} = \left(\|\mathbf{v}\|\|\mathbf{w}\|\sin\theta\right)\mathbf{n}$$

$$\mathbf{v}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & b & c \\ h & k & m \end{vmatrix} = (bm-ck)\mathbf{i}+(ch-am)\mathbf{j}+(ak-bh)\mathbf{k}$$

Component

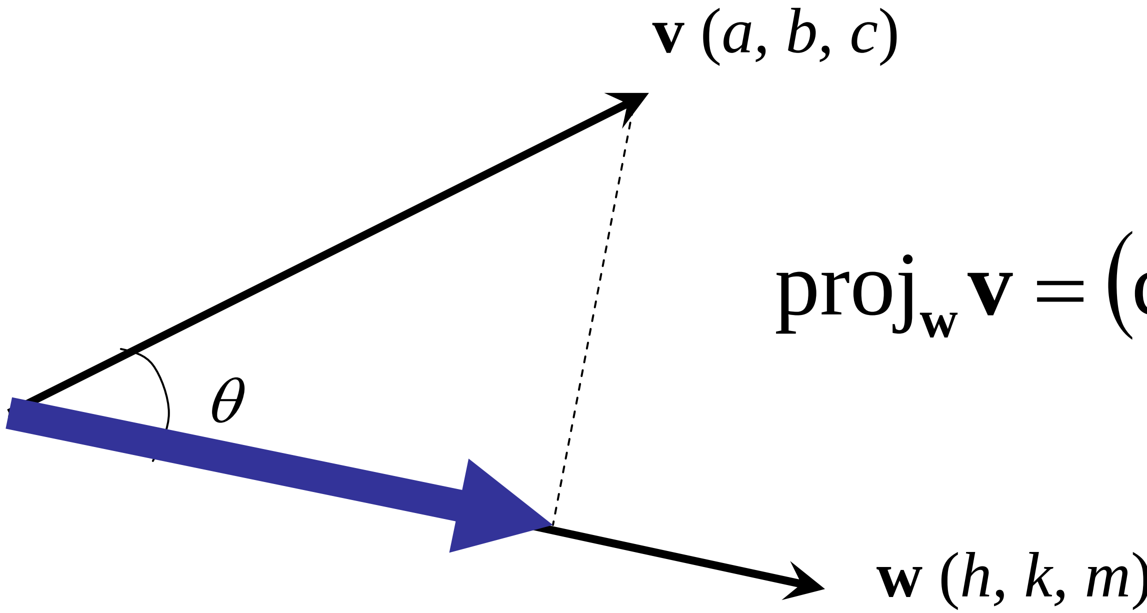


$$\text{comp}_{\mathbf{w}}\mathbf{v} = \|\mathbf{v}\|\cos\theta$$

$$\text{comp}_{\mathbf{w}}\mathbf{v} = \|\mathbf{v}\|\frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$$

$$= \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|}$$

Projection

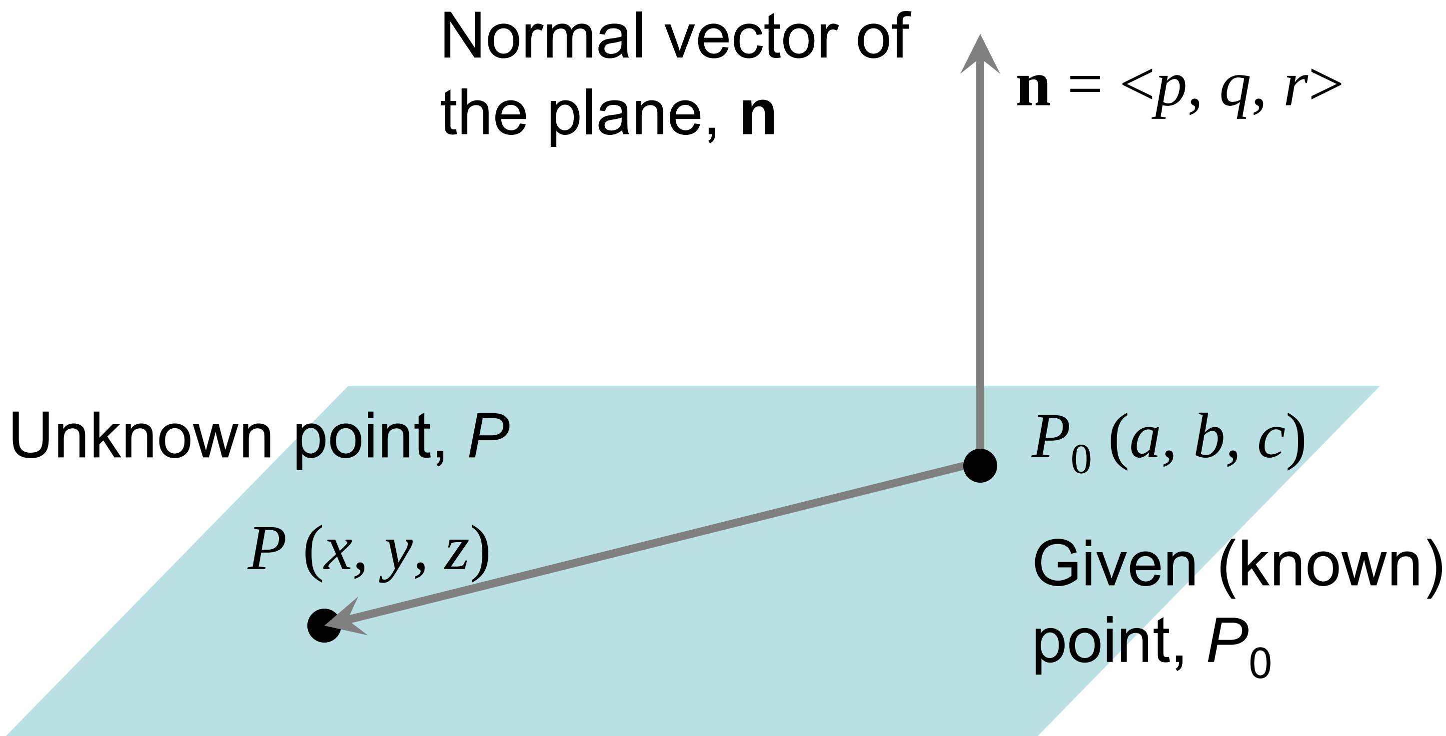


$$\text{proj}_{\mathbf{w}}\mathbf{v} = \left(\text{comp}_{\mathbf{w}}\mathbf{v}\right)\left(\frac{\mathbf{w}}{\|\mathbf{w}\|}\right)$$

$$\text{proj}_{\mathbf{w}}\mathbf{v} = \left(\frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|}\right)\left(\frac{\mathbf{w}}{\|\mathbf{w}\|}\right)$$

$$= \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$$

Plane equation



- Both points are within plane
- Both vectors are perpendicular

$$\overrightarrow{P_0P} \cdot \mathbf{n} = 0$$

Plane equation

$$\overrightarrow{P_0P} \cdot \mathbf{n} = 0$$

$$\langle (x-a),(y-b),(z-c) \rangle \cdot \langle p,q,r \rangle = 0$$

$$p(x-a)+q(y-b)+r(z-c)=0$$

$$px - pa + qy - qb + rz - rc = 0$$

$$px + qy + rz - pa - qb - rc = 0$$

$$px + qy + rz = pa + qb - rc$$