Baseballs and Boundary Layers

New Thoughts on Fastballs, Curveballs, and Other Pitches

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1 Introduction

Upon hearing that the curveball is

an optical illusion, Dizzy Dean remarked,

"Stand behind that tree, and

let me hit you with an optical illusion."

1.1 History

The heart of baseball lies in the ultimate showdown: pitcher versus batter. A typical pitch from a Major League pitcher arrives at the plate in less than .5 seconds. There are two pitches that are basic to baseball: the fastball and the curveball. Although these pitches are seen in everyday life, the physics behind their movements is still unclear.

The fastball is a pitch anyone can throw; simply throw the baseball. At the basic level, the fastball is a pitch that seems to move in a straight line on its way to the plate. There are two basic types of fastballs: the two seam and the four seam. For the two seam fastball, two seams rotate about the baseball for each revolution; likewise, the four seam fastball has four seams rotate for each revolution. The different seam orientations can give rise to different movements if thrown with enough spin rate. The two seam fastball can move away from or toward the batter; the four seam fastball does not. However, some hitters swear that a four seam fastball can rise on its way to the plate! Although this is improbable, there is an explanation for this perceived "rise". This paper will only deal with only four seam fastballs.

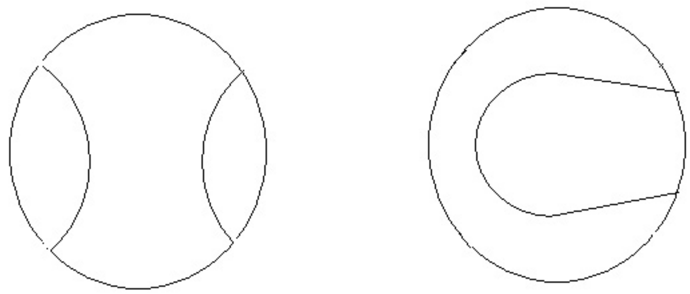

Figure 1: Two Seam and Four Seam Fastballs (viewed from above)

The two seam fastball is on the left; the four seam is on the right.

The curveball is introduced to many kids around 13 years old. Curveballs seem to change their path mid-flight. The first curve ball ever thrown is usually attributed to Candy Cummings around 1880. For much of half of the 20th century, scientists insisted that the curveball was an optical illusion. In 1959, however, Dr. Lyman J. Briggs proved the curveball actually "curves"[4]. However, the ball does not magically change its path mid-flight; it curves due to spin on the ball. It is the spin that causes the curveball to break[1].

The break on a baseball pitch is usually attributed to the Magnus Effect, after German engineer G. Magnus, who gave an explanation for the trajectory of spinning objects. The effect on a spinning sphere, such as a baseball, is known as the Robins Effect, who applied the Magnus Effect to spheres[9]. The phenomenon, however, is commonly referred to as the Magnus Effect. It is usually explained as a consequence of the Bernoulli Effect.

[1]In baseball terminology, the change in trajectory of the baseball is usually referred to as the "break". This term is used throughout the paper.

1.2 Bernoulli Effect

It is convenient to recall the Bernoulli Effect or Bernoulli Principle before proceeding. In its simplest terms, the pressure from a fluid on an object is inversely proportional to its kinetic energy. Hence, the slower the fluid is moving, the slower the kinetic energy and the larger the pressure. If there is a difference in fluid speed about an object, this rises to a net pressure on the object in the direction of the faster speed. This provides a force in that direction. For the case of the baseball, the spin causes the fluid to move more quickly on the side of the ball spinning with flow of the fluid (away from the direction of the baseball), creating a force in that direction. This is usually caused the Magnus Force.

1.3 Coordinates and Particulars

Throughout, a standard Cartesian coordinate system will be set up. The baseball will be projected along the horizontal (x) axis, vertical displacement will be about the vertical (z) axis. If the ball is moving left to right in relation to the page, the z axis will be straight up and the y axis will be into the page.

In this system, it should be noted that backspin spin of a baseball thrown to the right corresponds to counterclockwise motion as viewed on the page. Applying the right hand rule[2], it is found that the angular velocity vector is negative for this case. Similarly, a ball with topspin, clockwise motion, corresponds to positive angular velocity.

It is assumed the angular velocity of the baseball is constant during its flight path. To

[2]Same right hand rule is used to find currents induced by a magnetic field.

a good approximation this is true. The torque acting on the ball is slow down the spin is minimal; it is therefore neglected [2]

2 Necessary Fluid Dynamics

2.1 Reynolds Number

Every discussion of fluid mechanics must include the Reynolds number. It is the ratio of the inertia force and the viscous force acting on a body moving through the fluid. Reynolds number is given by:

$$Re = \frac{vd}{\nu} \tag{1}$$

where v is the velocity of the body, d is the characteristic length[3], and ν is the kinematic viscosity of the fluid.

For a baseball, which is typically pitched at speeds around 60-100 MPH in the Major Leagues, this corresponds to a range of Reynolds number of $4.1 * 10^5$ to $6.8 * 10^5$.

2.2 Navier-Stokes Equations

Much of fluid dynamics is characterized by equations that can not be analytically solved. The accepted equation that physicists use when dealing with fluids is the Navier-Stokes equation:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} + \frac{1}{\rho} \vec{F} \tag{2}$$

[3] For this paper, it is the diameter of a baseball.

where ρ is the density of the fluid, p is the pressure, $\vec{u}$ is fluid velocity, $\vec{F}$ is the force, and ν is the kinematic viscosity. For this paper, $\vec{u}$ is the same value that v has but is in the opposite direction.

In fluids, very few problems exists that can be solved by the above equation. However, it is useful to rewrite (2) as a unitless equation. This involves invoking scales for each variable and the equation becomes[4]:

$$\frac{\partial \vec{u'}}{\partial t'} + \vec{u'} \cdot \nabla' \vec{u'} = \nabla p' + \frac{1}{Re} \nabla'^2 \vec{u'} \tag{3}$$

where the prime designates the dimensionless variable. For simplicity, the force here is assumed to be zero.

This is useful because it shows the relationship of fluid problems on Reynolds number. Regardless of the scale, two situations that have the same Reynolds number have the same solutions. For example, to solve the problem of drag on a ship, one can model a ship using a scaled down ship, and the results will hold true on the larger scale.

As stated before, the Reynolds number is the ratio of the inertia force and the viscous force. For a steady flow, $\frac{\partial \vec{u'}}{\partial t'} = 0$ and the Reynolds number becomes:

$$Re \backsim \frac{\nabla'^2 \vec{u'}}{\vec{u'} \cdot \nabla' \vec{u'}} \tag{4}$$

In order for the forces to be dimensionally of the same order, the following condition must be true:

$$\delta \backsim \frac{ud^2}{\nu} \tag{5}$$

[4]This procedure is referred to a dynamical similarity. See Reference [10] for more information.

where δ specifies the boundary layer that forms as the flow goes around the sphere. For a baseball going 40 m/sec (about 90 MPH), this is

$$\delta \backsim \frac{ud^2}{\nu} = 1.7 \text{ mm} \tag{6}$$

The stitch height on a baseball is 1.8 mm (.07 in), which is of the order of the boundary layer[7]. This may play an important role in the separation of boundary layers. However, its role is beyond the scope of this paper.

3 Flow Past a Sphere

First, a baseball is generalized as a smooth sphere. There have been many studies that analyze the motion of smooth sphere and the resulting flow pattern. For purposes of generalization, it is also assumed that a sphere in a three dimensional flow can be approximated by a circular cylinder in two dimensional flow.

3.1 Non Rotating Smooth Sphere

Many solutions can be found for the Navier-Stokes equation for low Reynolds number. Probably the most famous of these solutions is the Stokes flow past a sphere, which is used to calculate drag on falling spheres. However, speeds typical during a baseball game yield large Reynolds numbers. It is because of this large speed that boundary layers become important and give the baseball its unique properties.

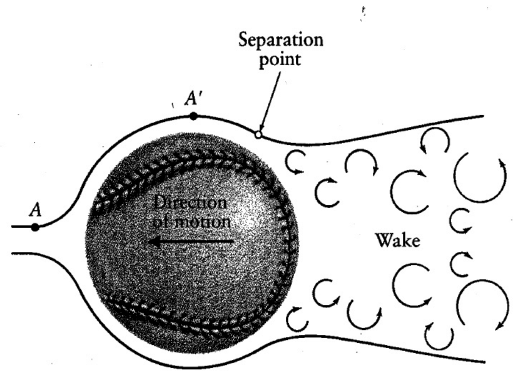

Figure 2: The Trailing Wake on Non-Rotating Baseball

For a large enough Reynolds number, the boundary layer can become turbulent. For a baseball, this threshold lies well beyond the possible velocities for a baseball[6]. Therefore the boundary layer is laminar, and therefore will separate more easily[10].

The boundary layers separate from the ball, leaving a trailing wake. In Figure 2, the separation point is shown for a non-rotating sphere. Although, the picture is of a baseball, the result holds true for a smooth sphere as well (see Reference [6]).

It is interesting to note that at smaller Reynolds numbers ($Re < 1$), the flow will produce the well known Karman Vortex street. For $Re << 1$, no trailing vortices will be produced. Notice in Figure 3 as the Reynolds number increases, the number of wakes produced is increased.

3.2 Rotating Smooth Sphere

A rotating sphere has long been known to curve as it is projected. Sir Isaac Newton was the first to recognize this in the late 1600's. In Newtons description, the larger drag on the

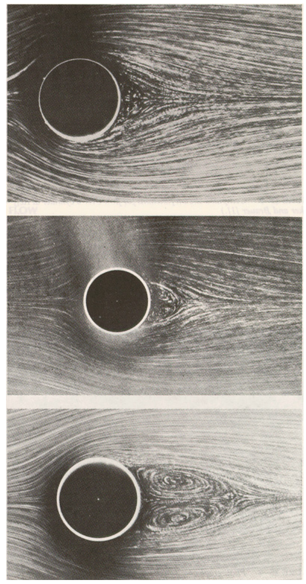

Figure 3: Flow Patterns at lower Reynolds numbers

Progressing down the figure corresponds to Reynolds number $<< 1$, $\backsim 1$, > 1.

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third-base side translates to a larger force or a lower pressure and the ball swerves toward the first base side of home plate. This was the simplest explanation of what happened to a curveball, but was not completely correct.

Additionally, others noted this, including Magnus. Magnus used the Bernoulli effect in his theory on the spinning ball He argued that: "A spinning ball induces in the air around it a kind of whirlpool of air in addition to the motion of air past the ball as the ball flies through the air"[8]. This circulating air slows down the flow of air past the ball on one side, and speeds it up on the other side. From Bernoulli's theorem, the ball experiences a force on the low-pressure (high-speed) side; therefore, the ball changes its straight line path.

However, this understanding is incomplete. The more complete work was by Prandtl

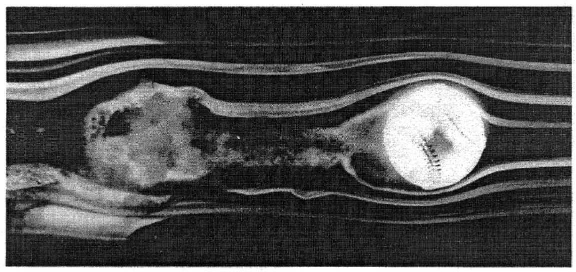

Figure 4: Flow Diagrams for a Non-Rotating Sphere

Figure 5: Flow Diagrams for a Rotating Sphere

in 1904, who introduced the Prandtl layer around a object that is rotating. This layer asymmetrically separates, causing a turbulent jet in the flow around the object. This causes a change in the objects direction.

The idea of Prandtl layer separation is best seen through an illustration, Figures 4 and 5. Because of the rotation, the separation point changes; it no longer separates near the top of the ball. The separation point is rotated around the ball, dependent on the spin direction.

4 Baseball, Rotating Sphere

The derivation of forces used here are used by Alaways [2] et. al. to find the force on the baseball. While this works for many cases, it fails to describe what *causes* the forces. Nevertheless, it is useful to describe the procedure. It is this procedure that most closely resembles the Magnus force, and the usual explanation of the movement of a baseball.

In a standard Cartesian coordinate system, the baseball is projected in a manner such that its translational velocity, $\vec{v}$, is along the x axis. For the purposes of this paper, we will neglect any initial angle with which the ball is projected.

Although a baseball can be projected with a spin about (almost) any axis, here the spin, $\vec{\omega}$, will be restricted to be about the y axis for a fastball and curveball. This leads to:

$$\vec{v} = v_x \hat{x} \quad \text{and} \tag{7}$$

$$\vec{\omega} = \omega_y \hat{y} \tag{8}$$

Of course, this implies that:

$$\vec{\omega} \times \vec{v} = \omega_y v_x \hat{z} \tag{9}$$

Either case, the fastball or the curveball, the baseball will be projected with four seams.

4.1 Drag

The drag on the baseball is not of significance to this study, but it is proportional to v^2 and is measurable. It is given in the usual form of:

$$\vec{D} = -\frac{1}{2}\rho C_D A v \vec{v} \tag{10}$$

where A is the cross sectional area, ρ is the density of the fluid, and C_D is the drag coefficient. The drag force is a retarding force and opposes the motion.

4.2 Lift Force

As stated earlier, the Magnus force does not completely explain the movement of the baseball. However, it does work experimentally. For the baseball projections that are dealt with here, the magnus force will produce a lift force given by

$$\vec{L} = \frac{\rho C_L A v^2}{2} \frac{\vec{\omega} \times \vec{v}}{|\vec{\omega} \times \vec{v}|} \tag{11}$$

where C_L is the lift coefficient, A is the cross sectional area, and v is the translational velocity of the ball.

Often in fluid texts, the above equation is written with $\vec{v} \times \vec{\omega}$, where the velocity is the velocity of the fluid. It bears repeating that the velocity here is the velocity of the baseball, hence the cross product reads as it does[2].

4.3 Cross Force

There is an additional force acting on the baseball, called the cross force. It is included due to the asymmetric pattern of the baseball seams. It is included by Alaways to find a better determination of the lift coefficient. The cross force, Y is given by:

$$\vec{Y} = \frac{1}{2}\rho C_Y A v^2 \frac{\vec{L} \times \vec{D}}{|\vec{L} \times \vec{D}|} \tag{12}$$

where ρ, A, and v are the same and C_Y is the cross force coefficient.

It is defined in this manner so that the cross force is perpendicular to both the drag force and the lift force. Since the drag force on a baseball is simply a retarding force and is fairly well understood, some combination of the lift force and the cross force must explain the break of a baseball. Therefore, the Magnus force, which is usually attributed to the break of the baseball, must be some combination of the lift force and cross force.

4.4 Accelerations in Each Direction

From Newton's second law, the net force on the baseball is given by:

$$\vec{F}_{net} = \vec{L} + \vec{D} + \vec{Y} \tag{13}$$

Using Equations (10) to (12) and (13) each term is expanded into its x,y, and z components:

$$\dot{v}_x = \beta[\frac{C_L}{\omega}(v_z\omega_y - \omega_z v_y) - C_D v_x + \frac{C_Y}{\omega V}(v_y^2\omega_x - v_x v_y \omega_y - v_x v_z \omega_z + v_z^2\omega_x)] \tag{14}$$

$$\dot{v}_y = \beta[\frac{C_L}{\omega}(v_x\omega_z - \omega_x v_z) - C_D v_y + \frac{C_Y}{\omega V}(v_z^2\omega_y - v_y v_z \omega_z - v_y v_x \omega_x + v_x^2\omega_y)] \tag{15}$$

$$\dot{v}_z = \beta[\frac{C_L}{\omega}(v_y\omega_x - \omega_y v_x) - C_D v_z + \frac{C_Y}{\omega V}(v_x^2\omega_z - v_z v_x \omega_x - v_z v_y \omega_y + v_y^2\omega_z)] - g \tag{16}$$

where

$$\beta \equiv \frac{\rho A v}{2m}$$

and g is the acceleration due to gravity.

In order to fully satisfy these differential equations and solve for position, it is also noted that the first time derivative of each position direction (x, y, z) gives the corresponding velocity in that direction (v_x, v_y, v_z). This equations are quite tedious to go through and are included here for completeness. This paper will refer mostly to the Equations (10) to (12) and (13).

5 Different Pitches

5.1 Fastball

A fastball is thrown with backspin, i.e. negative angular velocity. The ball will be projected in the x-direction, and its angular velocity will be $-\omega_y \hat{y}$.

Of course, this implies that:

$$\vec{\omega} \times \vec{v} = -\omega_y v_x \hat{z} \tag{17}$$

In other words, the force terms that contain $\vec{\omega} \times \vec{v}$ (only the lift force contains this) will have components only in the z direction. For both the fastball and curveball, the additional velocity in the z direction that the ball picks up due to gravity is neglected; from Equations (14) to (16) it adds a force that is proportional to the z component of the velocity. The force is in the x direction; it is a retarding force in addition to the drag.

Alaways[2] noted that the cross product does not play a major role in four seam fastballs due to the symmetry of seam orientation. Therefore, this paper will assume the break on a fastball is only due to the lift force. Again, it is noted the drag force is a retarding force, and while it is important, does not lead to unexpected breaks in the trajectory of the baseball. Therefore, this paper will neglect the cross force on a fastball. Also, to analyze just the break on the fastball, this paper also neglects the drag.

Therefore, the only force acting on the baseball is the lift force, and it is entirely in the positive z direction.

Using the lift part of Equation (16), we get

$$\begin{aligned} \dot{v}_z &= \beta \frac{C_L}{\omega}(-(-\omega_y v_x)) - g \\ &= \frac{\rho A V}{2m}\frac{C_L}{\omega}(\omega_y v_x)) - g \end{aligned} \tag{18}$$

Therefore, the ball has force that is counteracting gravity.

Typical values for a fastball include a velocity of 40 m/sec (about 90 MPH), an angular velocity of 30 rps, and a mass of .142 kg. The density of air, ρ, is 1.2 kg/m^3. For this situation, C_L can be determined from Figure (6), and is seen to be approximately .2 for pitchers[5][2].

Therefore Equation (18) becomes:

$$\begin{aligned} \dot{v}_z &= \frac{\rho A v}{2m}\frac{C_L}{\omega}(\omega_y v_x)) - g \\ &= 5.633 \text{ m/sec}^2 - 9.8 \text{ m/sec}^2 \\ \dot{v}_z &= -4.167 \text{ m/sec}^2 \end{aligned}$$

Although the fastball drops because of the negative acceleration, it does not fall as

[5]Alaways data is from collegiate and semi-professional pitchers only. This lift coefficient is not necessarily the same as it would be for a Major League pitcher. However, there is a close resemblance in the types of pitches one sees on the collegiate level and the Major League level. One major difference is the amount of spin that is imparted to the ball. One can assume that Major League pitchers can impart a greater spin, and therefore change its trajectory a greater amount, than their collegiate counterparts.

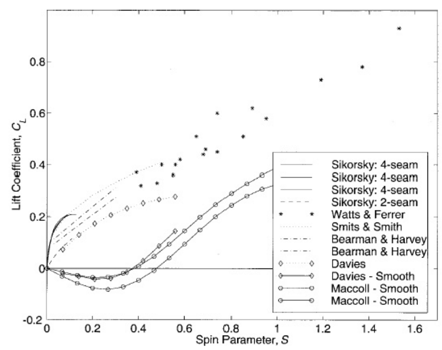

Figure 6: Variation of Lift Coefficient of Spin

fast as it would under the influence of gravity alone.

The angular velocity drops out explicitly in this case, which is a very peculiar result. However, it is implicitly contained in the lift coefficient, maintaining an observed dependence of the acceleration and subsequent displacement on angular speed.

5.1.1 The Split Fingered Fastball

The split finger baseball is gripped with a wide grip and is thrown with backspin (like a regular fastball). This pitch is characterized by its "drop" as it approaches the batter. The wide grip on the split finger leads to small spin rate (about 10 rps) for the ball.

According to Figure 6, as the spin decreases relative to the velocity, the lift coefficient also decreases. As the spin goes to zero, the lift coefficient also goes to zero. Therefore, the split fingered fastball has more of a pronounced drop as it nears the plate, as if there was no lift acting on it.

Of course, a more detailed analysis may show that the split fingered fastball may actually experience a downward force in addition to gravity. If this is the case, the baseball resembles a smooth sphere. When the spin rate goes below a critical point, the sign of the lift reverses[10]. For a split finger fastball, the spin rate lies in this region. More detailed analysis is needed to verify the parameters of the split fingered fastball.

5.1.2 The Rising Fastball

Many players believe a fastball thrown with enough speed and spin will actually rise as it nears the plate. In order for the ball to actually rise, it most overcome gravity. Mathematically, $\dot{v}_z$ should be positive. Using Equation 18,

$$\begin{aligned}
0 &< \dot{v}_z = \frac{\rho A v}{2m}\frac{C_L}{\omega}(\omega_y v_x)) - g \\
g &< \frac{\rho A v}{2m}\frac{C_L}{\omega}(\omega_y v_x)) \\
g &< (.0018\text{m}^{-1}) * C_L v_x^2 \\
556.734\ \text{m}^2/\text{sec}^2 &< C_L v_x^2 \\
\frac{556.734\ \text{m}^2/\text{sec}^2}{v_x^2} &< C_L
\end{aligned}$$

For a typical speed such as 40 m/sec, this would only be true if omega was very large, of the order of 660 rps.[6] This result has not been duplicated in any baseball game to date and is highly improbable.

Therefore, the rising fastball is simply a myth. In fact, because human brains expect

[6] Since C_L should be about .4, this leads to a spin parameter of about .6 from Figure 6. This leads to an omega of about 660 sec^-1

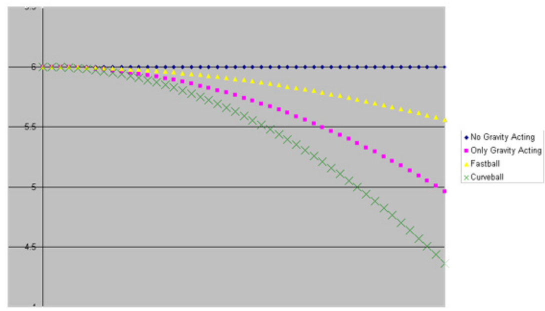

Figure 7: Qualitative Graph for Ball Trajectory of Different Pitches

These results do not include drag. It is assumed the results from the drag force would be approximately the same. For pitches that are in the air longer, the drag force would have more time to act, and therefore play more of a role in the drop of the ball.

things to follow the path of gravity, this may explain why the fastball seems to rise. Looking at the Figure 7, under only the influence of gravity, the ball would fall about 1 m on its way to the plate. With spin, the ball only falls about .5 m. It seems as if the ball "rises" from its gravity path.

5.2 Curveball

A curve ball is thrown with topspin, i.e. positive angular velocity. Therefore, the analysis is the same as it was for the fastball; simply replace $-\omega_y$ with $+\omega_y$.

Again, the only force that plays a significant role is the lift. The curve is also thrown with four seams, so the same argument concerning the cross force applies; the cross force is negligible.

Using the lift part of Equation (16), we get

$$\begin{aligned}\dot{v}_z &= \beta\frac{C_L}{\omega}((-\omega_y v_x)) - g \\ &= \frac{\rho A V}{2m}\frac{C_L}{\omega}(-\omega_y v_x)) - g\end{aligned}$$

Using the same values for the fastball, Equation 19 becomes:

$$\begin{aligned}\dot{v}_z &= \frac{\rho A V}{2m}\frac{C_L}{\omega}(-\omega_y v_x)) - g \\ &= -5.633 \text{ m/sec}^2 - 9.8 \text{ m/sec}^2 \\ &= -15.433 \text{ m/sec}^2\end{aligned}$$

The acceleration is greater than g in the downward direction. This ball breaks heavily downward, as it is seen in practice. Overhand curveballs do indeed break more downward than other pitches.

It should be noted that curveballs are usually not thrown with a velocity of 40 m/sec. A curve ball velocity is more of the order of 35 m/sec (80 MPH). However, the C_L is still approximately the same[2]. The slower speed means the ball will be arriving later and the force has longer to act. This implies the break will more pronounced.

5.3 Slider

A batter facing a well thrown slider sees a dot on the ball as it approaches him. This dot implies that the ball is rotating about this axis. From a right handed pitcher, the ball has an angular velocity vector that points in the positive x direction, i.e. $\vec{\omega} = \omega_x \hat{x}$. Sliders are known to break horizontally, with some downward movement. Again, the ball will be thrown in the positive x direction.

Here, the cross force is not neglected; it must be this force that gives rise to the horizontal movement of the ball. The actual value of the drag force is not important to this calculation; it is simply acknowledged to retard the path of the path, so again it does appear in the calculations.

The lift force is taken from Equation 11. It is proportional to the cross product of $\vec{\omega}$ and $\vec{v}$. As the ball travels through the air, it will begin to accelerate in the downward z direction, and therefore pick up velocity in that direction. Using the velocity of the ball to be therefore $\vec{v} = v_x\hat{x} - v_z\hat{z}$, the equations of motion 14 to 16 yield:

$$\dot{v}_x = \beta[\frac{C_Y}{\omega V}(v_z^2\omega_x)]$$

$$\begin{aligned} \dot{v}_y &= \beta[\frac{C_L}{\omega}(-\omega_x v_z)] \\ &= +\beta[\frac{C_L}{\omega}(\omega_x v_z)] \end{aligned}$$

$$\dot{v}_z = \beta[\frac{C_Y}{\omega V}(-v_z v_x \omega_x)] - g$$
$$= +\beta\frac{C_Y}{\omega V}v_z v_x \omega_x)] - g$$

The result in the x direction is of little consequence for this paper; there is an additional force that retards the motion in the x direction. It is not important to our studies.[7]

Also, the result in the z direction is somewhat uninteresting. Here there is a force that counters gravity. This is seen in practice. Sliders do not seem to acceleration downward as much as other pitches.

The major result from this calculation is in the y direction. Although the force in the y direction does not arise from the cross force term, there is a force in the +y direction. Indeed, sliders from a right handed pitcher break away from a right handed batter, in the positive y direction.

As an aside, if a baseball game was played on a planet with no gravity, there would be no force in the z direction, and therefore no gain in velocity in the z direction. If the ball was projected with a velocity only in the x direction, the force in the y direction would vanish ($v_x = 0$). Sliders so not exist in a no gravity environment. As a corollary, sliders have smaller breaks on places such as the moon, where acceleration due to gravity is smaller.

[7] In fact, for the fastball, this paper only included a passing mention of this.

6 Another Description of the Break on a Curveball

The Alaways model seems to accurately model the break of a several pitches of a baseball. However, there is one glaring omission. There is no hint of the reason WHY the ball breaks in the manner it does. After all, this is the main question surrounding a baseball.

6.1 Numerical Simulation

The nearest explanation of why the ball moves as it does, and the resulting physics comes from a mathematician. In 2000, Joey Huang [5] presented a numerical solution for the Navier-Stokes equation and Newton's second law for the curveball.

He noted that the spin of the ball must be able to change the behavior of the fluid around it, implying the fluid must be viscous[5].

He set up the Navier Stokes equations in moving coordinates with the ball, so that

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} \tag{19}$$

The motion of the fluid around the ball is the important factor in determining the path. The ball is spinning, so it is necessary to also include vorticity, Γ, in the calculations:

$$\Gamma = \nabla \times \vec{u} \tag{20}$$

This also satisfies

$$\frac{\partial \Gamma}{\partial t} + (\vec{u} \cdot \nabla)\Gamma = \nu \nabla^2 \vec{\Gamma} \tag{21}$$

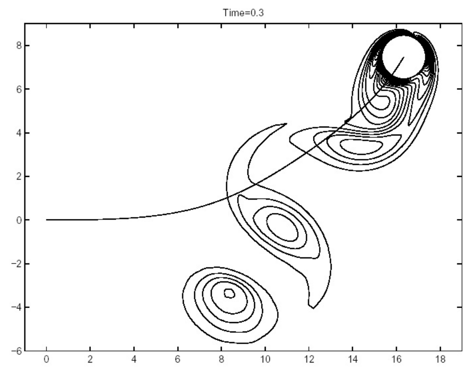

Figure 8: Trajectory of the Ball and Contour Plot of Velocity[5]

Changing to polar coordinates in two directions:

$$Re^{s+i\theta} = x + iy$$

Equation 21 becomes:

$$\frac{\partial\Gamma}{\partial t} + \frac{\frac{\partial\psi}{\partial s}\frac{\partial\Gamma}{\partial\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial\Gamma}{\partial s}}{R^2e^{2s}} = \nu\nabla^2\vec{\Gamma} \tag{22}$$

where ψ is the stream function. Using this, the flow around the baseball can be computed. However, to find the movement of the ball, pressure on the ball must also be found. In order to find the equations of motion, Huang used stress tensors and Fourier modes. It is beyond the scope of this paper to include that, but it is fruitful to mention it can be done. Of course, this can not be solved analytically, but a numerical solution can be found and plotted. Figure 8 shows a qualitative analysis. In this plot, Huang did not use baseball data; he simply produced a plot showing the effects of his analysis. The result models the path of a curveball.

7 Boundary Layers

Even with this, however, there is no mention of when the boundary layers separate. It is clear from the simulation, the a separation of boundary layer corresponds to a change in the trajectory of the ball. From experimental results, it is clear that boundary layers play an important role in the break of a spinning sphere.

Boundary layers sperate to form a trailing wake behind the baseball. For a non-rotating baseball, this wake is directly behind the ball. However, the spin on the ball deflects the wake. For a fastball like spin, the wake is deflected downward, and this provides a upward force on the ball. Since the wake has a negative momentum change, the baseball must have a positive momentum change. It is clear the force on the ball arises from the separation of boundary layers and its effect on the ball.

In Figure 8, notice where the fluid is rotating, i.e. where the contour is closed. These are the trailing wakes as seen in 9 points that the boundary layer separated from the ball. Where there is a wake, the trajectory of the ball changed. Towards the end of the plot, more wakes have been shed, creating a larger break on the ball. Each wake "pushes" the ball more. This corresponds to a larger force. It is the shedding of the boundary layer and the resulting wakes that ultimately causes the ball to break. This is ultimately what causes the Magnus force.

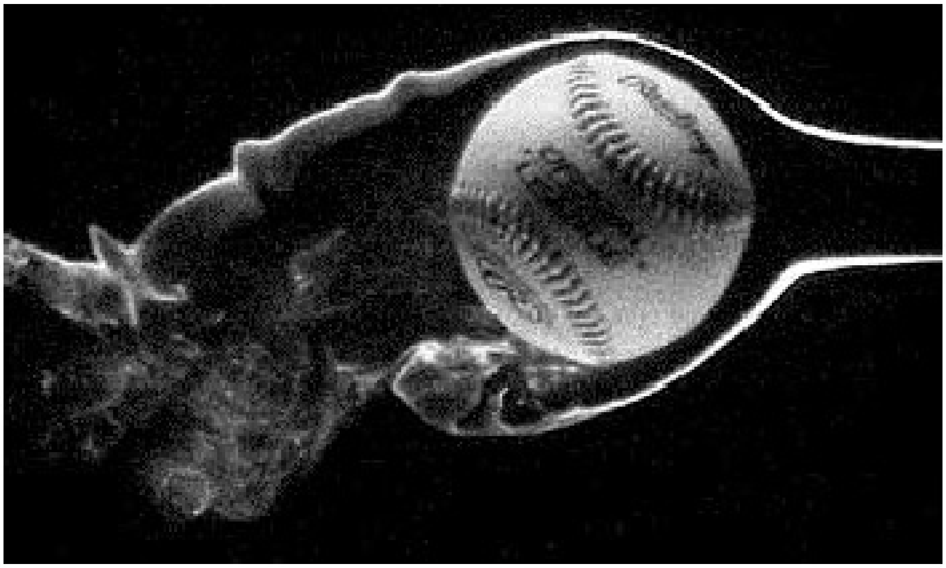

Figure 9: Boundary Layer Separation on the Resulting Wake From a Spinning Fastball

The wake is deflected down; therefore the ball is deflected up.

8 Conclusions

An eminent physicist once noted, "There are two problems that interest me deeply. The first is the unified field theory; the second is why does a curveball break. I believe that, in my lifetime, we may solve the first, but I despair the second." It is the complicated nature of the Navier-Stokes equation that leads to this difficulty. Coupled with the asymmetric seams on a baseball, this is a rather difficult problem.

Several models have been proposed to solve the problem of the trajectory of a baseball. The Magnus force, while archaic, is the often used explanation for the break of a baseball. However, what actually causes the force is usually glazed over by providing some explanation of the Bernoulli Effect. A somewhat updated form of the Magnus force, using lift, drag, and cross forces, seem to accurately describe the motion. Yet, it does not explain why these forces arise.

A numerical simulation of the corresponding Navier-Stokes equation yields positive results, yet does not explain the orientation of stitches in relation to the break of a baseball. Additionally, it fails to extrapolate to explain other pitches besides the curveball.

The ultimate solution will involve boundary layer separation. The Magnus force arises from the separation of boundary layers. The spot on the ball where the boundary layer separates seems to lead to the break of the ball. The boundary layers leave a wake, and the ball feels a net force coming from the place the boundary layer separates and forms a wake. This is ultimate reason why the curveball breaks.

The next step for physicists is to incorporate a more precise understanding of boundary layers into the break of a baseball. It is obvious different stitch orientations affect the break of the ball, as does the direction of angular velocity. These factors must be included in the next step to solve the problem of the trajectory of a baseball.

9 Other Results

The Alaways derivation of lift force and its subsequent dependence on seams and angular velocity can be used to model the baseball's flight. Other studies done seem to contradict this, namely Watts and Ferrer[11]. Their study shows that seam orientation does not matter to the deflection of the baseball. However, their data was taken at relatively low speeds, i.e. 17.9 m/sec (about 40 MPH). This leads to the conclusion that at low speeds, the orientation of seams is inconsequential. This helps explain the lack of huge breaks on a baseball during Little League games, where the speeds usually top out at 40 MPH. As the kids get older, the speed at which they can project and spin the baseball increases; therefore, the possible break on the baseball does as well. Once the kids reach about 13 years old, where speeds are of the order of 60 MPH, batters face bigger changes in the trajectory of a baseball.

Also, many batters swear the ball breaks sharply as it nears them. Alas, this is only a mind trick. Take the simplest case: a constant force on the baseball. The constant force yields a constant acceleration. This leads to a displacement of the ball that goes as $\frac{1}{2}at^2$. In other words, the break of the ball is a parabola in time, leading to the result that ball breaks as a parabola as it nears the batter, and does not posses a sharp break in its displacement.

In a more complicated manner, other studies have proven this. Using strobe photography and high speed analysis, Allman[3] found results that proved the break on a baseball follows an arc, not a sharp break.[2].

However, in even the simplest model, a constant force, half of the deflection from a straight line path occurs in the last 5 meters in the baseball's path to the plate[1]. Indeed, the ball seems to "break" as it nears the plate.

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11 Other Readings

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